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EE-606: Solid State Devices
Lecture 3: Elements of Quantum Mechanics

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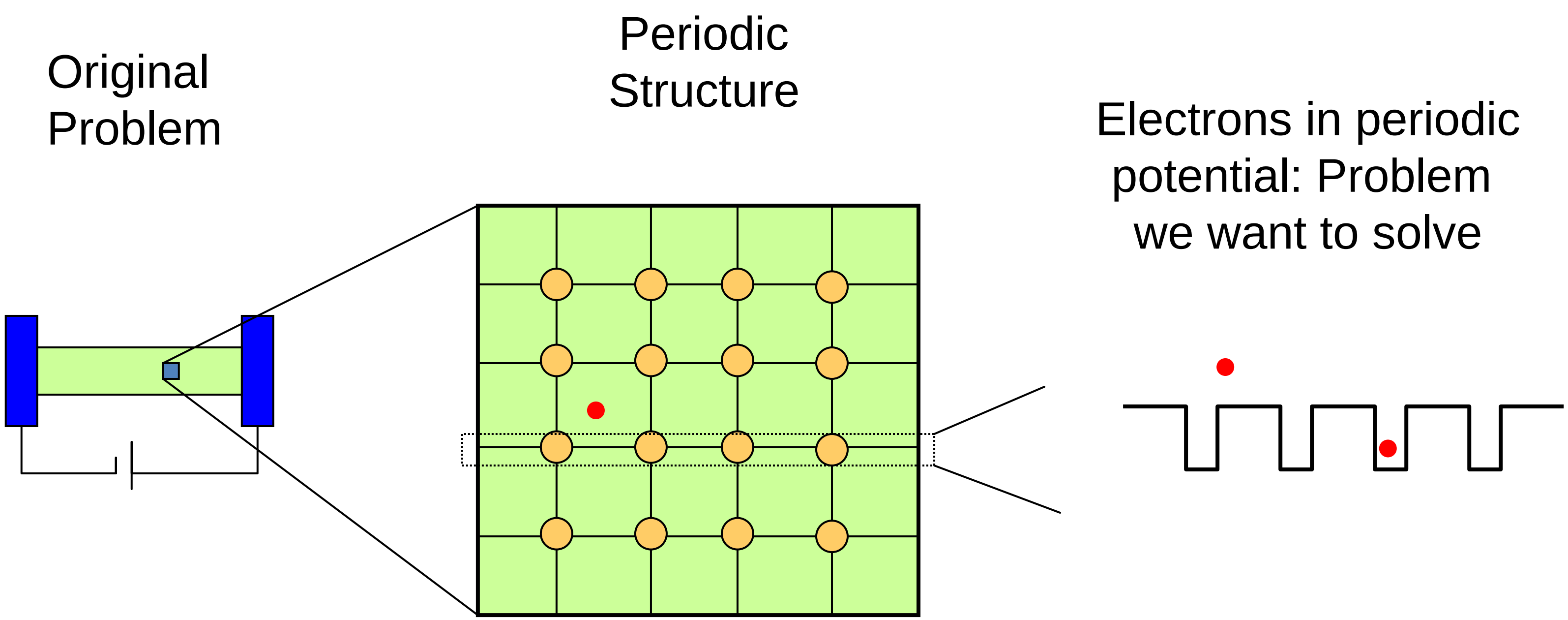
Outline

1) **Why do we need quantum physics**
2) Quantum concepts
3) Formulation of quantum mechanics
4) Conclusions

Reference: Vol. 6, Ch. 1 (pages 23-32)

Do I really need Quantum Mechanics ?



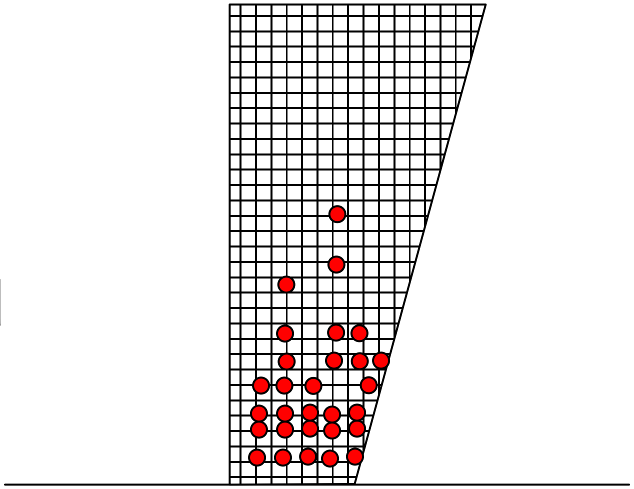
If it were large objects, like a skier skiing past a set of obstacles, Newton's mechanics would work fine, but in a micro-world ……

Carrier Density

Carrier number = Number of states x filling factor

Chapters 2-3 — Chapter 4

Total number of occupants
= Number of apartments
X The fraction occupied



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1) Why do we need quantum physics
2) **Quantum concepts**
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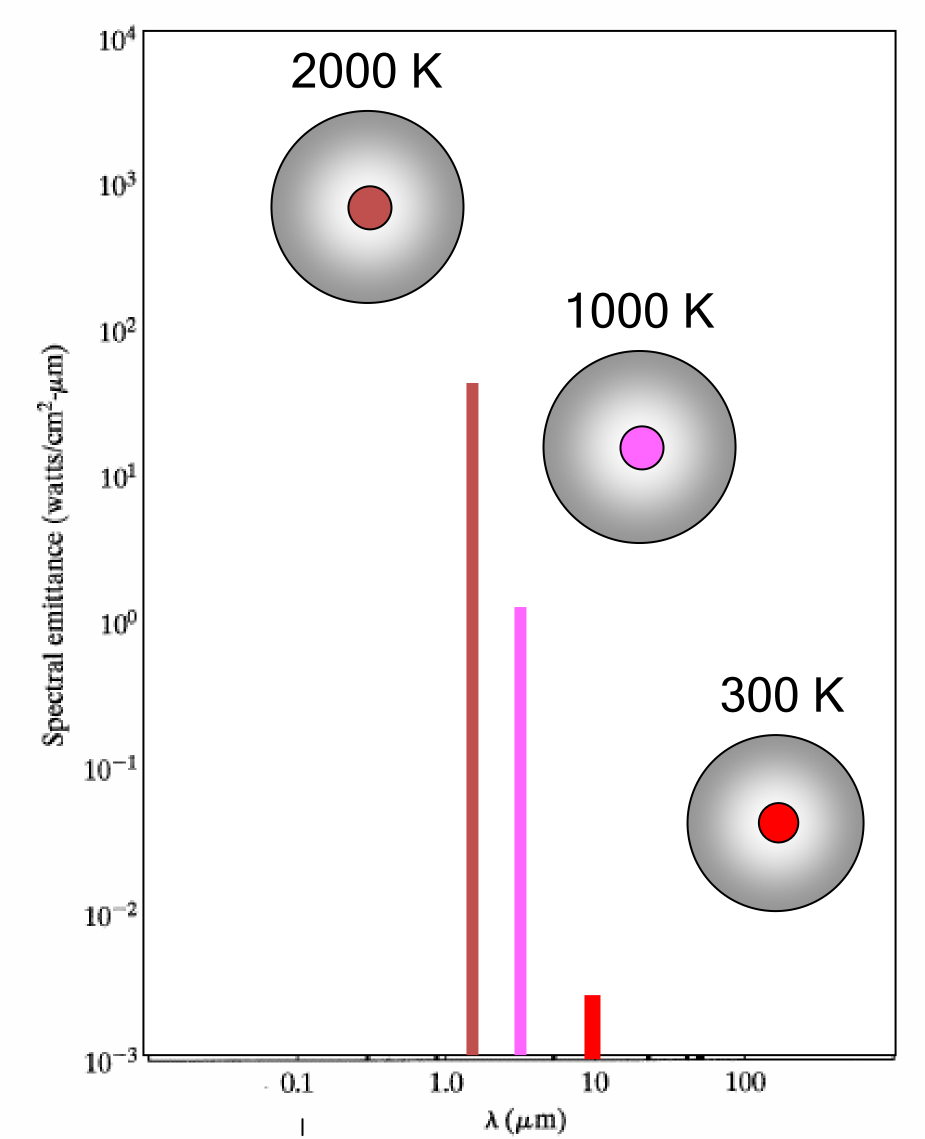
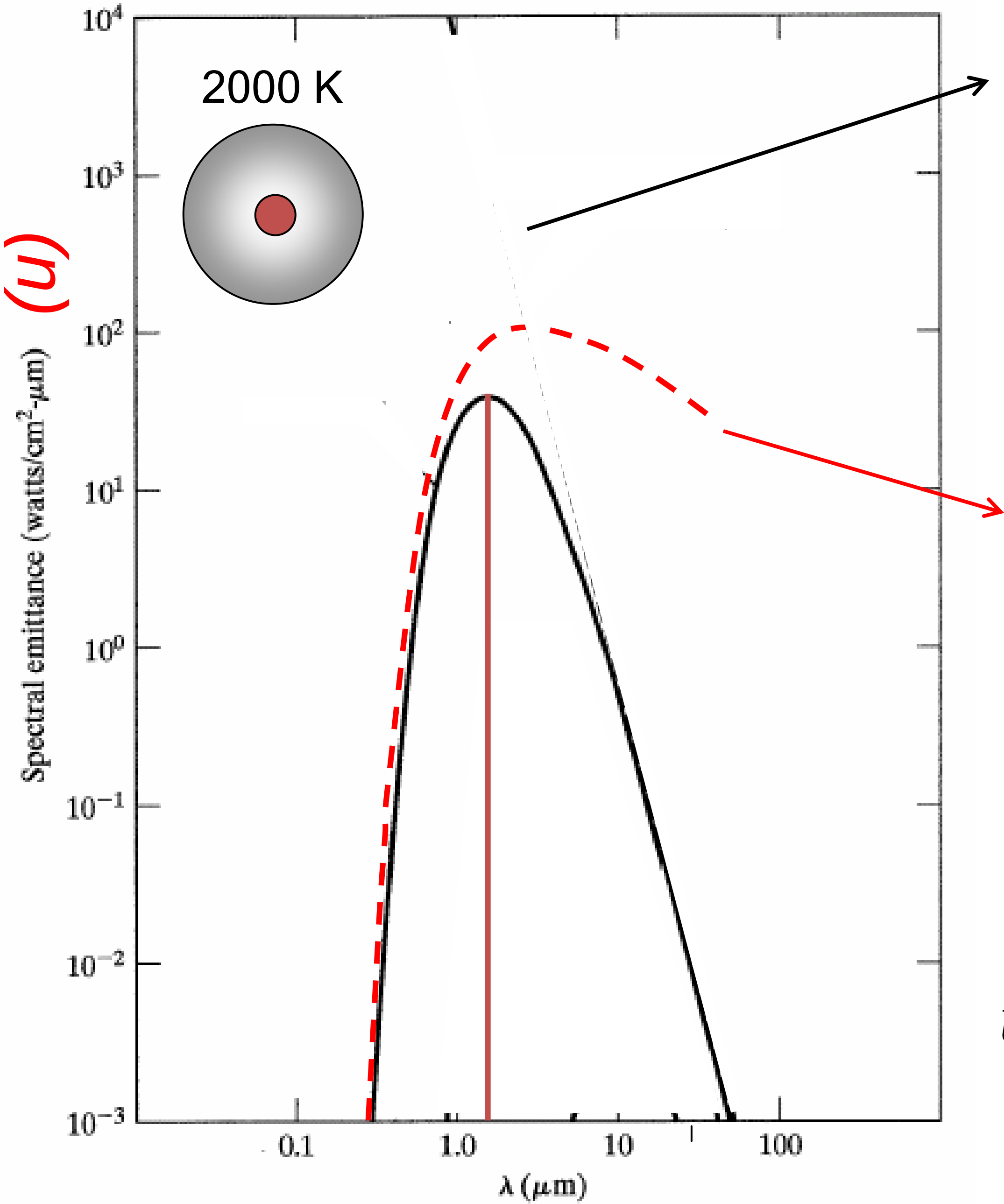
Four Quantum Concepts ..

- Blackbody Radiation
- Photoelectric Effect
- Bohr Atom
- Wave Particle Duality

(1) black-body radiation





Rayleigh-Jeans Formula

$$u(\lambda, T) \propto k_B T / \lambda^4$$

$$\log(u) = -4\log(\lambda) + \log(T)$$

Wein's Formula

$$u \propto \frac{e^{-\beta/\lambda T}}{\lambda^5}$$

Plank's fitting formula

$$u(\lambda, T) \propto \frac{1}{\lambda^5}\left[\frac{1}{e^{\beta/\lambda T} - 1}\right]$$

Interpretation of Plank's Formula

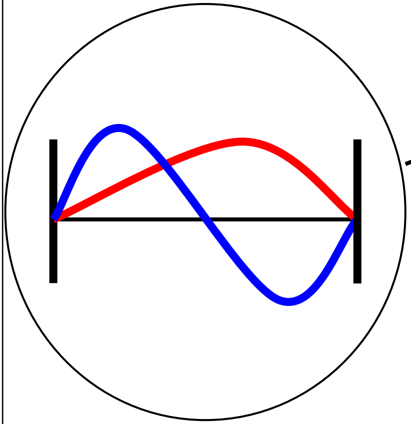
$$u(f,T) = u(\lambda,T)\frac{d\lambda}{df} \sim \frac{1}{\lambda^5}\left[\frac{1}{e^{\beta/\lambda T}-1}\right]\frac{d\lambda}{df} \qquad \lambda = \frac{c}{f}$$

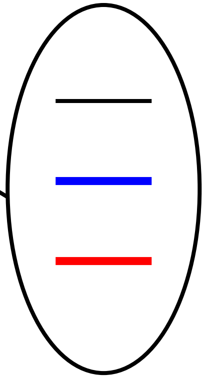
$$\sim f^2 \times hf \times \left(\frac{1}{\mathrm{e}^{hf/kT}-1}\right)$$

nos. of modes

Energy of mode

Occupation Probability

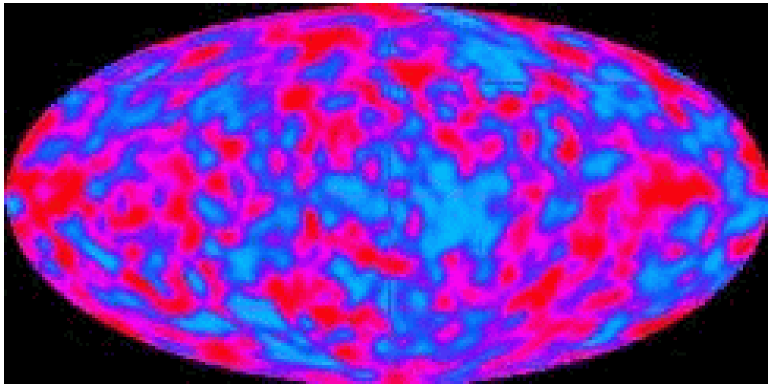




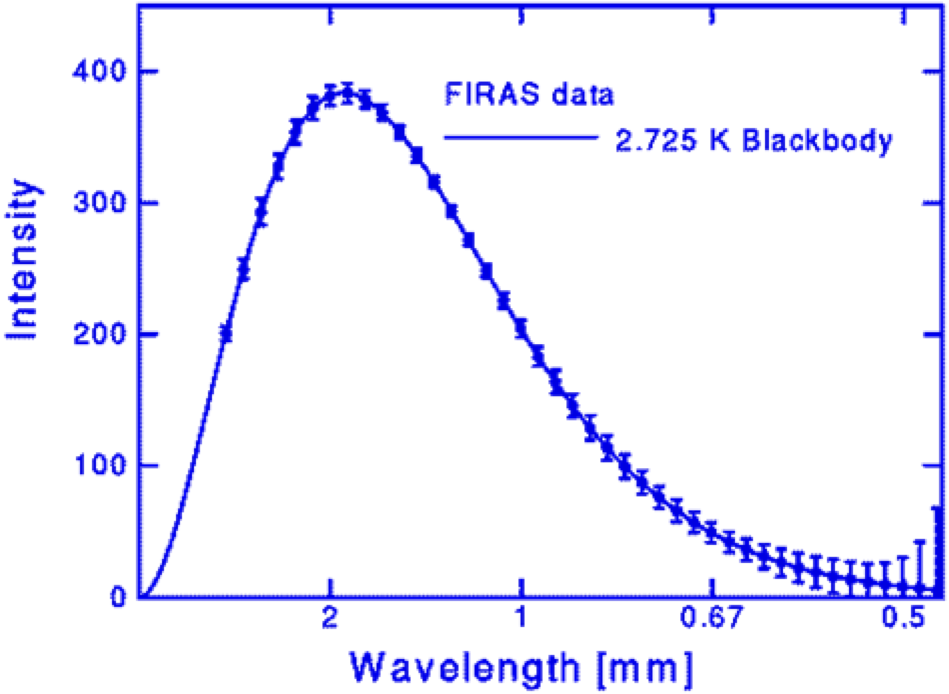
EM emission occurs in discrete quanta of

$$E = hf \qquad \text{n=1,2, ... N}$$

Recent Example: COBE Data

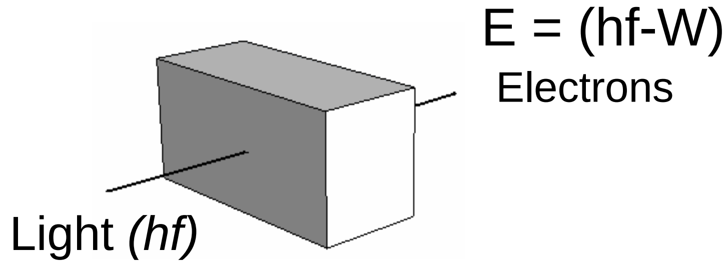


J.C. Mather, Astrophysics J., 1990.

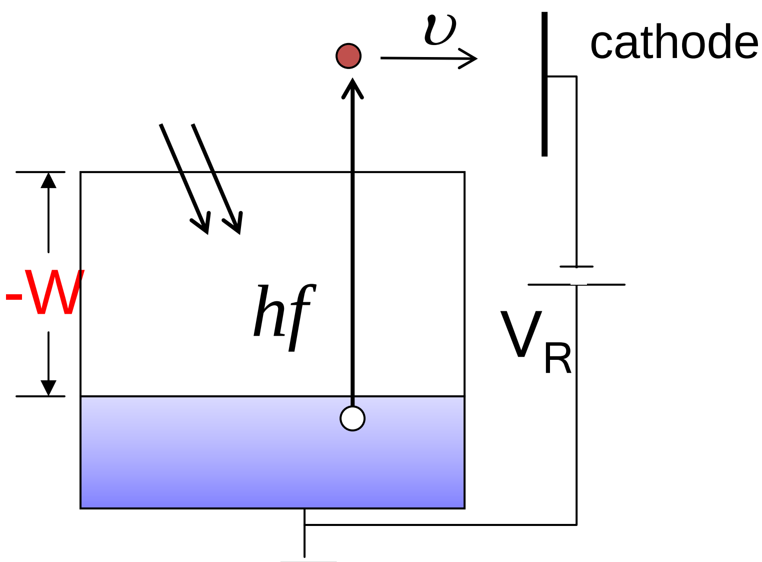


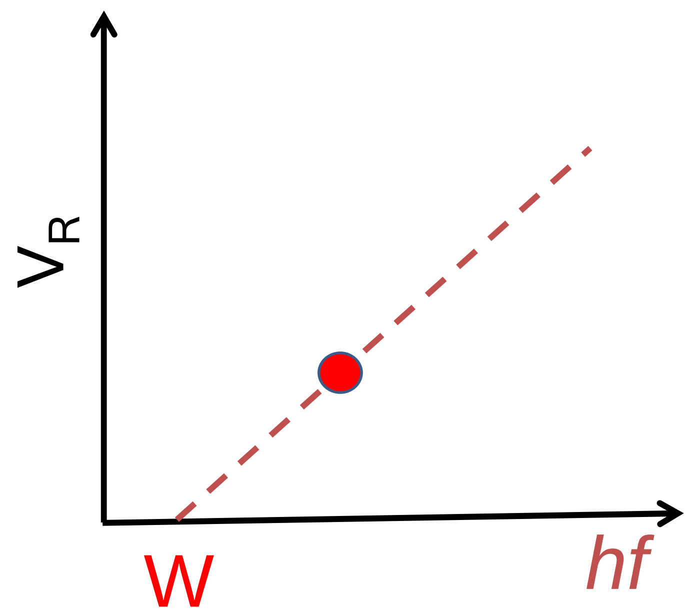
Show that the cosmic background temperature is approximately 3K. Can you "see" this radiation?

(2) Photoelectric Effect



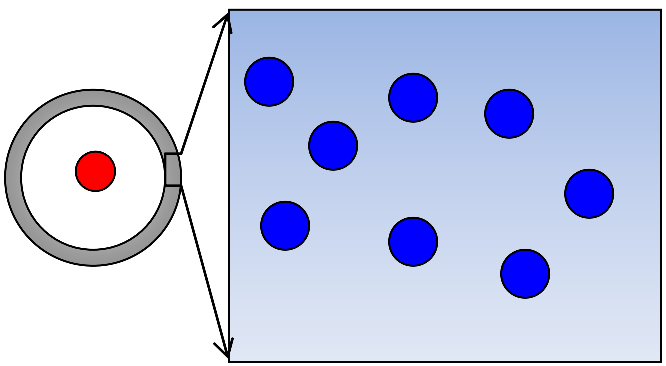
$$qV_R \approx \left(\frac{1}{2}\right) m_0 \upsilon^2 = hf - W$$

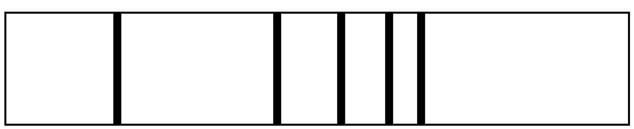




Absorption occurs in quanta as well, consistent with photons having ***E=hf***

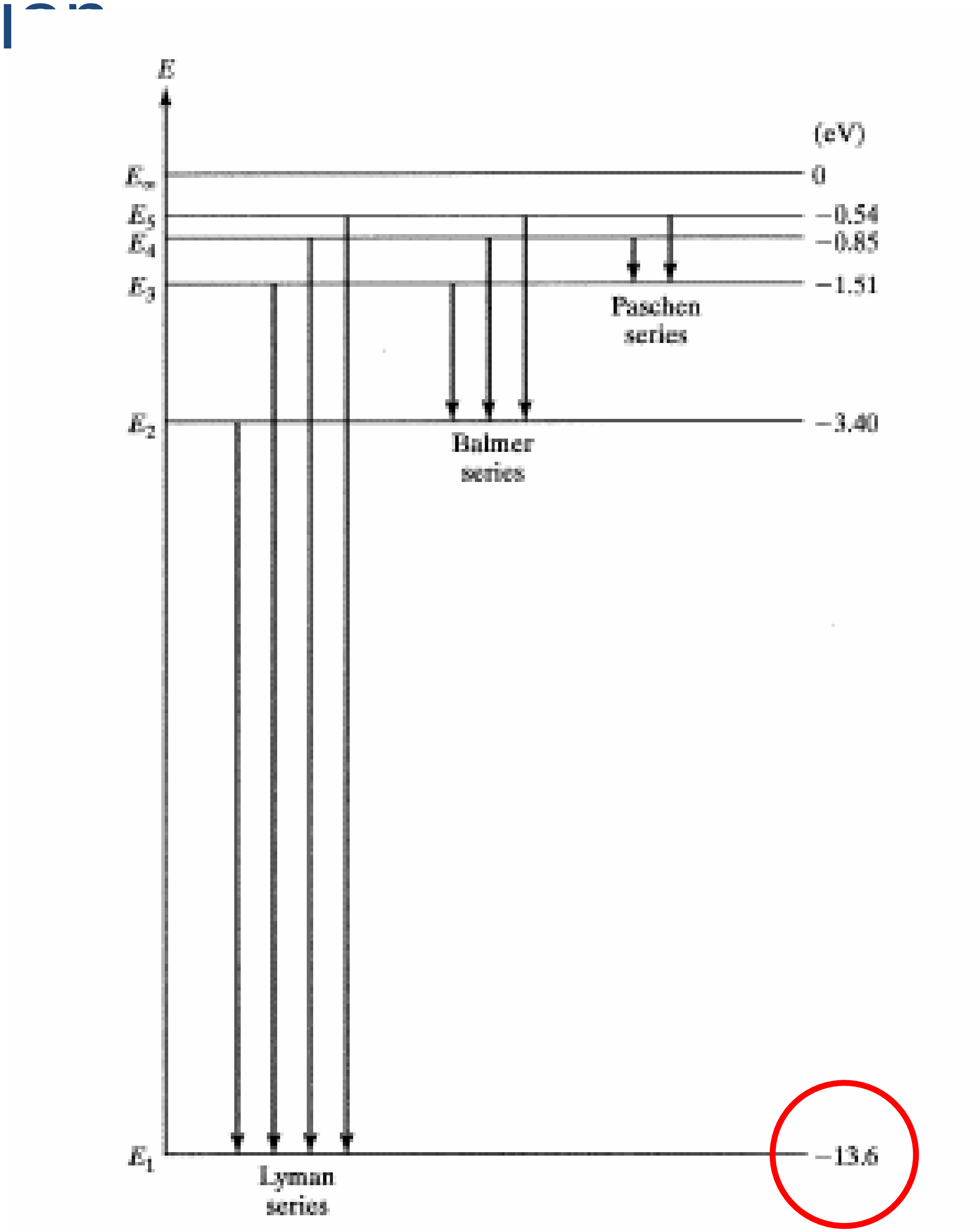
Origin of Quantization





frequency

$$E_{m,n} = const \times \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$$



(3) Bohr Atom ..

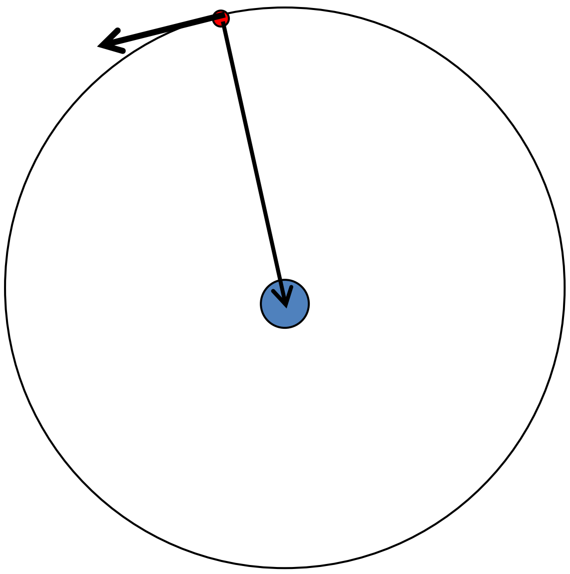
Assume that angular momentum is quantized:

$$L_{\mathbf{n}} = m_0 v r_{\mathbf{n}} = \mathbf{n}\hbar \qquad \mathbf{n} = 1, 2, 3, \ldots$$

$$\upsilon = n\hbar / m_0 r_n$$

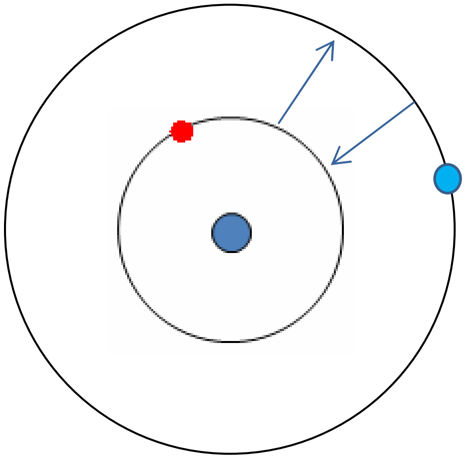
$$\frac{m_0 v^2}{r_{\mathbf{n}}} = \frac{q^2}{4\pi\varepsilon_0 r_{\mathbf{n}}^2}$$

$$r_{\mathbf{n}} = \frac{4\pi\varepsilon_0 (\mathbf{n}\hbar)^2}{m_0 q^2}$$



(3) Bohr Atom (continued) …

$$r_{\mathbf{n}} = \frac{4\pi\varepsilon_0(\mathbf{n}\hbar)^2}{m_0 q^2}$$



$$\text{K.E.} = \frac{1}{2}m_0 v^2 = \frac{1}{2}(q^2/4\pi\varepsilon_0 r_{\mathbf{n}})$$

$$\text{P.E.} = -q^2/4\pi\varepsilon_0 r_{\mathbf{n}} \qquad (\text{P.E. set} = 0 \text{ at } r = \infty)$$

$$E_{\mathbf{n}} = \text{K.E.} + \text{P.E.} = -\frac{1}{2}(q^2/4\pi\varepsilon_0 r_{\mathbf{n}})$$

$$\boxed{E_{\mathbf{n}} = -\frac{m_0 q^4}{2(4\pi\varepsilon_0 \mathbf{n}\hbar)^2} = -\frac{13.6}{\mathbf{n}^2}\,\text{eV}}$$

$$E_{m,n} = const \times \left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

(4) Wave-Particle Duality

Photons act both as wave and particle, what about electrons ?

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

$$hf = pc$$

m_0=0 (photon rest mass)

$$p = hf / c$$

$$= h / \lambda \quad \text{(because } c = \lambda f \text{)}$$

$$= \hbar k \quad \text{(because } k = 2\pi / \lambda \text{)}$$

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3) **Formulation of Schrodinger Equation**
4) Conclusions

Schrodinger Equation for electrons

$$E = \sqrt{m_0^2 c^4 + p^2 c^2} \approx m_0 c^2 \left[1 + p^2 c^2 / 2 m_0^2 c^4 + \ldots \right]$$

$$E - m_0 c^2 \quad = \quad V \; + \; (p^2 / 2 m_0)$$

$$hf = \hbar\omega \quad = \quad V \; + \; (\hbar^2 k^2 / 2 m_0)$$

Schrodinger Equation (continued)

$$\hbar\omega \quad = \quad (\hbar^2 k^2 / 2m_0) + V$$

Assume, $\Psi(x,t) = A\exp(-i(\omega t - kx))$

$$d\Psi / dt = -i\omega\Psi \quad \text{and} \quad d^2\Psi / dx^2 = -k^2\Psi$$

$$i\hbar\frac{d\Psi}{dt} = \left(-\frac{\hbar^2}{2m_0}\frac{d^2\Psi}{dx^2}\right) + V\Psi$$

Conclusions

1. Given chemical composition and atomic arrangements, we can compute electron density by using quantum mechanics.

2. We discussed the origin of quantum mechanics – experiments were inconsistent with the classical theory.

3. We saw how Schrodinger equation can arise as a consequence of quantization and relativity, but *this is not a derivation*.

4. We will solve some toy problems in the next class to get a feeling of how to use quantum mechanics.