Hydrodynamical Simulations of Laser Interaction with Targets

Research and International Cooperation

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International Cooperation

- **Los Alamos National Laboratory, USA – numerical methods for Lagrangian and ALE hydrodynamics**
- **CELIA (Centre Lasers Intenses et Applications), University Bordeaux, France – numerical methods for Lagrangian and ALE hydrodynamics; modelling of laser interaction with targets**
- **IPPLM (Institute of Plasma Physics and Laser Microfusion), Warsaw, Poland – modelling of laser interaction with targets**
- **Utsunomia University, Japan**
- **CEA (Commissariat a l'energie atomique et aux energies alternatives), Saclay, France**
- **LULI (Laboratoire d'Utilisation des Lasers Intenses), Ecole Polytechnique, Polaiseau, France**
- **Advanced Photonics Research Institute, Gwangju Institute of Science and Technology (GIST), Gwangju, Korea**

Overview

- numerical treatment of advection equation and conservation laws
- Euler equations
- motivation example for Lagrangian formulation
- hydrodynamical model with heat conductivity and laser absorption
- numerical methods used in our PALE (Prague ALE) code
  - hyperbolic part – Arbitrary Lagrangian Eulerian (ALE) method
  - parabolic part – heat conductivity
  - laser absorption – source term in internal energy equation

- **laser plasma application, which cannot be treated by pure Lagrangian method**
  - **high velocity impact problem**
  - **double foil target**
  - **foam target**
  - **jet formation by annular laser profile**

Advection Equation

- **advection (one-side wave) equation** $u(x,t)$

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0, \qquad u_t + au_x = 0$$

  with initial condition $u(x,0) = u_0(x)$ **has solution**

$$u(x,t) = u_0(x - at)$$

- **continuum area of independent variables** $(x,t) \in R \times (0,\infty)$ **is replaced by computational grid** $(x_j, t_n) = (j\Delta x, n\Delta t), j \in Z, n \in N_0$
- **continuum function** $u(x,t)$ **is replaced by discrete grid function** $u_j^n \approx u(x_j, t_n)$
- **simple difference scheme**

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} + a\frac{u_j^n - u_{j-1}^n}{\Delta x} = 0$$

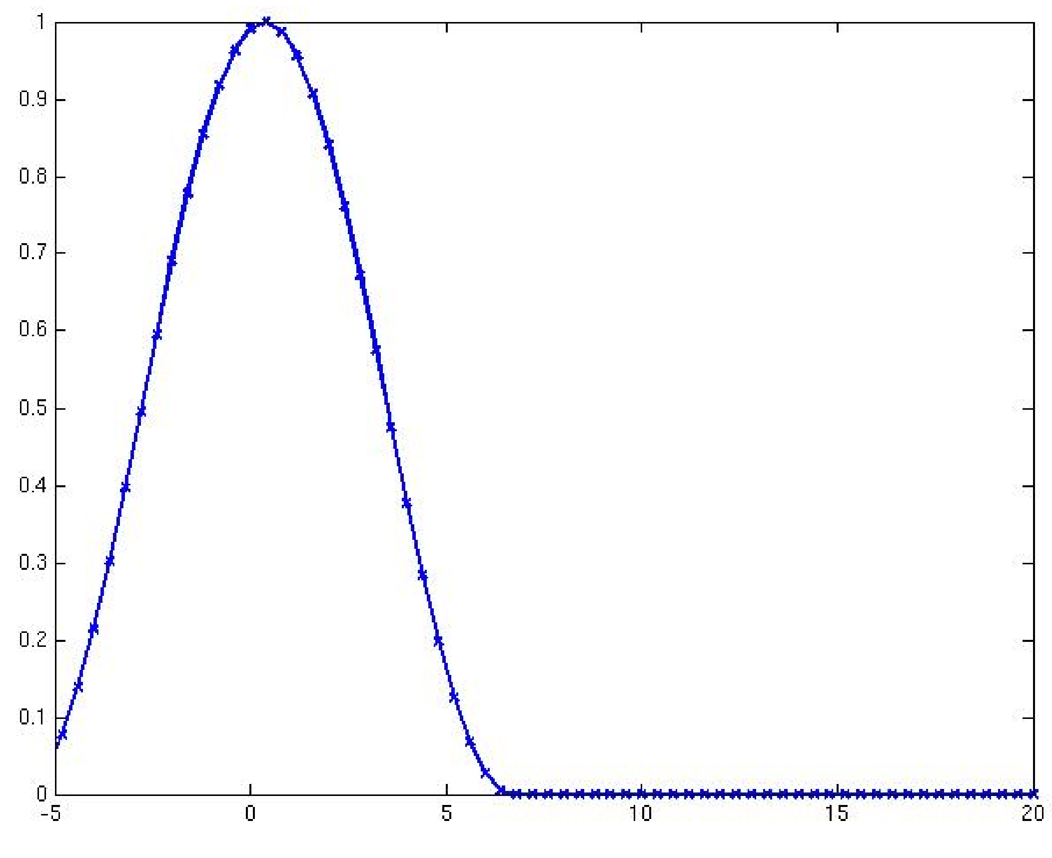
- **from time level** n **we compute new time level** $n+1$

$$u_j^{n+1} = u_j^n - \frac{a\Delta t}{\Delta x}(u_j^n - u_{j-1}^n)$$

Advection equation – numerical solution

- **advection equation with initial condition and with** $a = 1$

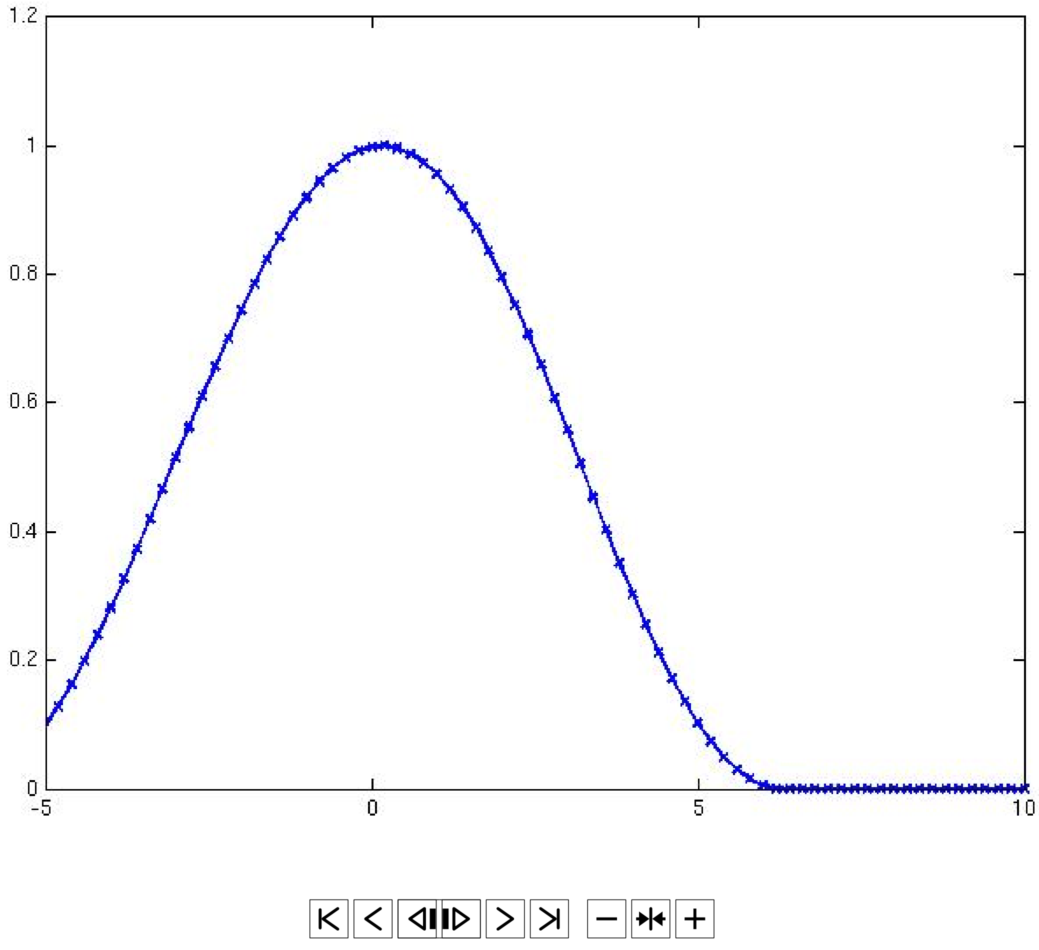
$$u_t + a u_x = 0, \quad u(x,0) = u_0(x) = \begin{cases} \frac{1+\cos(x/2)}{2} & \textbf{pro} \quad |x| < 2\pi \\ 0 & \textbf{jinak} \end{cases}$$



Burgers equation

- **Burgers equation with initial condition**

$$u_t + uu_x = 0, \quad u(x,0) = u_0(x) = \begin{cases} \frac{1+\cos(x/2)}{2} & \textbf{pro} \quad |x| < 2\pi \\ 0 & \textbf{jinak} \end{cases}$$



Conservation Laws

- **Burgers equation**

$$u_t + uu_x = 0, \quad u_t + \left(\frac{u^2}{2}\right)_x = 0$$

can have discontinuous solution

- **discontinuity - shock wave - special numerical methods**

- **general conservation law – system** $U_t + (f(U))_x = 0$

- **three types of simple waves**
  - **shock wave**
  - **contact discontinuity**
  - **rarefaction wave**

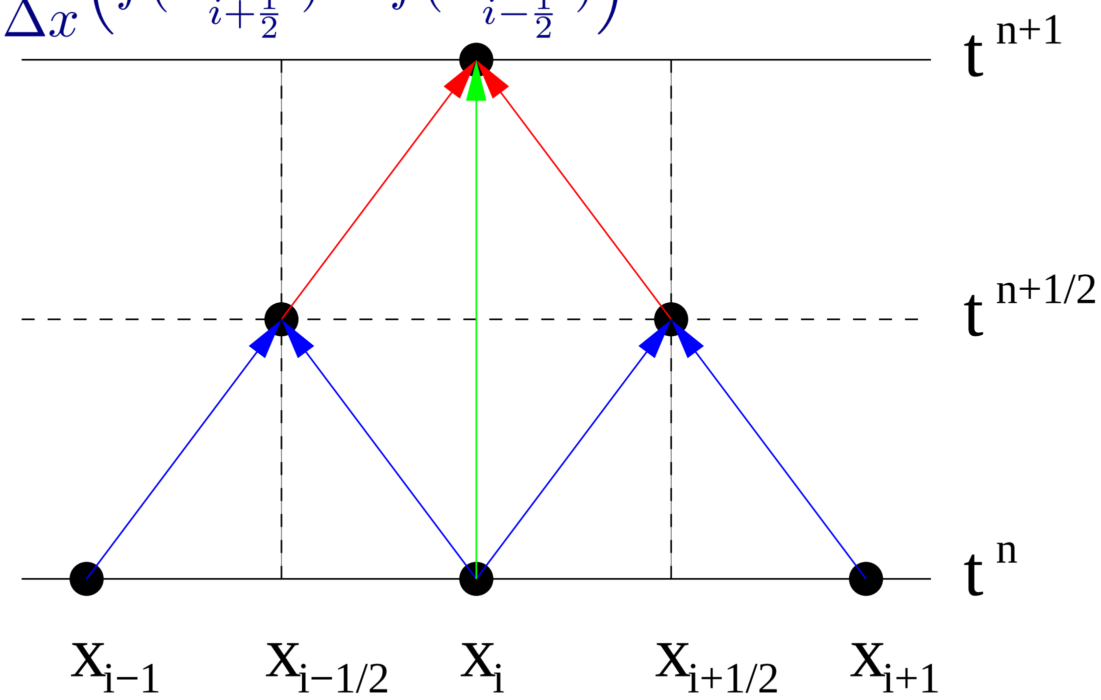
Composite Schemes for Conservation Laws

- **conservation law** $U_t + f(U)_x = 0$
- **Lax-Friedrichs scheme, diffusive, two step variant**

$$U_{i+\frac{1}{2}}^{n+\frac{1}{2}} = \frac{1}{2}(U_i^n + U_{i+1}^n) - \frac{\Delta t}{2\,\Delta x}\Big(f(U_{i+1}^n) - f(U_i^n)\Big)$$

- **Lax-Wendroff scheme, simple fluxes, dispersive**

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x}\Big(f(U_{i+\frac{1}{2}}^{n+\frac{1}{2}}) - f(U_{i-\frac{1}{2}}^{n+\frac{1}{2}})\Big)$$



- **composite scheme LW LF** k

Euler Equations

- **Euler equations in 3D**

$$\begin{pmatrix} \rho \\ \rho\,u \\ \rho\,v \\ \rho\,w \\ E \end{pmatrix}_t + \begin{pmatrix} \rho\,u \\ \rho\,u^2 + p \\ \rho\,u\,v \\ \rho\,u\,w \\ u\,(E+p) \end{pmatrix}_x + \begin{pmatrix} \rho\,v \\ \rho\,u\,v \\ \rho\,v^2 + p \\ \rho\,v\,w \\ v\,(E+p) \end{pmatrix}_y + \begin{pmatrix} \rho\,w \\ \rho\,u\,w \\ \rho\,v\,w \\ \rho\,w^2 + p \\ w\,(E+p) \end{pmatrix}_z = 0$$

equation of state for ideal gas

$$E = \frac{p}{\gamma - 1} + \frac{1}{2}\rho\left(u^2 + v^2 + w^2\right)$$

- **basic equations for hydrodynamical modelling of plasma**

Motivation for Lagrangian Formulation

- **laser plasma is created by laser interaction with targets**
- **target is $0.8\mu m$ thin Aluminum foil; Prague Asterix Laser System (PALS) laser at 3-rd harmonics $\lambda = 438\,\mathrm{nm}$, pulse duration $250ps$, focus $40\mu m$, energy $200J$; animation**
- **computational mesh is fixed to the fluid and moves with the fluid**
- **no mass flux between cells through edges**
- **computation domain changes with time**
- **problems with large changes of computational domain volume and/or shape (compression or expansion**

)
- **naturally treated moving boundaries**
- **typically used in laser plasma simulations**

Euler Equations in Lagrangian Coordinates

- **Lagrangian coordinates move together with the fluid**

$$\rho \frac{dU}{dt} = \mathrm{div}\, \mathbf{F}(U)$$

- $\mathrm{d}/\mathrm{d}\,t = \partial/\partial\, t + \mathbf{u} \cdot \mathrm{grad}$ **with velocity** $u = (u, v, w)$ **is the total Lagrangian time derivative including convective terms**

$$U = \begin{pmatrix} \eta \\ \mathbf{u} \\ E \end{pmatrix}, \mathbf{F} = \begin{pmatrix} \mathbf{u} \\ -pI \\ -p\mathbf{u} \end{pmatrix}$$

- $\eta = 1/\rho$ **is the specific volume and** I **is the unit matrix**
- **ideal gas equation of state**

$$p = (\gamma - 1)\rho\varepsilon, \quad \varepsilon = E - \frac{\mathbf{u}^2}{2}, \quad c^s = \sqrt{\frac{\gamma p}{\rho}}$$

- **eigenvalues of flux Jacobian matrix are** $0, \pm c^s$
- **Lagrangian particle movement by** $d\mathbf{X}/dt = \mathbf{u}$

Staggered Lagrangian method in 1D

- **scalars ρ, ε, p in cells $i+1/2$; vectors u, x in nodes i**
- **equations for velocity and internal energy**

$$\rho\frac{du}{dt} = -p_x, \qquad \rho\frac{d\varepsilon}{dt} = -pu_x$$

- **scheme for velocity and internal energy**

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = -\frac{p_{i+1/2}^n + q_{i+1/2}^n - p_{i-1/2}^n - q_{i-1/2}^n}{m_i}$$

$$\frac{\varepsilon_{i+1/2}^{n+1} - \varepsilon_{i+1/2}^n}{\Delta t} = -(p_{i+1/2}^n + q_{i+1/2}^n)\frac{\frac{1}{2}(u_{i+1}^{n+1} + u_{i+1}^n) - \frac{1}{2}(u_i^{n+1} + u_i^n)}{m_{i+1/2}}$$

- **artificial viscosity q added to p in compressed cells**

$$q_{i+1/2}^n = \begin{cases} 0 & , u_{i+1}^n - u_i^n \geq 0 \\ -\frac{3}{2}\rho_{i+1/2}^n(u_{i+1}^n - u_i^n)\sqrt{(\gamma-1)\gamma\varepsilon_{i+1/2}^n} & , u_{i+1}^n - u_i^n < 0 \end{cases}$$

- **mesh, density**

$$\frac{x_i^{n+1} - x_i^n}{\Delta t} = \frac{u_i^{n+1} + u_i^n}{2}, \qquad \varrho_{i+1/2}^{n+1} = \frac{m_{i+1/2}}{x_{i+1}^{n+1} - x_i^{n+1}}$$

Cell-centered Lagrangian method in 1D

- **all quantities in cells [Despres et al. 2005][Maire et al. 2007]**
- **conservative equations** $\rho\frac{dU}{dt} = \mathbf{F}(U)_x$ **for** $U = (\eta, u, E)$ **with fluxes** $\mathbf{F} = (u, -pI, -pu)$
- **the simplest scheme**

$$\frac{U^{n+1}_{i+1/2} - U^{n}_{i+1/2}}{\Delta t} = \frac{F^*_{i+1} - F^*_i}{m_{i+1/2}}$$

- **fluxes given by the approximate acoustic Riemann solver**

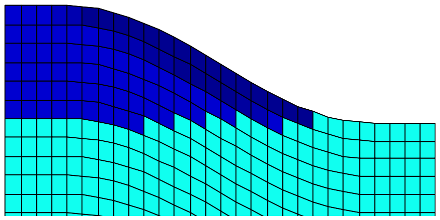
$$\begin{aligned} u^*_i &= \frac{z^n_{i+1/2}u^n_{i+1/2} + z^n_{i-1/2}u^n_{i-1/2}}{z^n_{i+1/2} + z^n_{i-1/2}} - \frac{p^n_{i+1/2} - p^n_{i-1/2}}{z^n_{i+1/2} + z^n_{i-1/2}}, \\ p^*_i &= \frac{z^n_{i+1/2}p^n_{i-1/2} + z^n_{i-1/2}p^n_{i+1/2}}{z^n_{i+1/2} + z^n_{i-1/2}} - \frac{z^n_{i+1/2}z^n_{i-1/2}}{z^n_{i+1/2} + z^n_{i-1/2}}(u^n_{i+1/2} - u^n_{i-1/2}), \end{aligned}$$

the impedance $z^n_j = \rho_j c^s_j$ **with the speed of sound** c^s_j

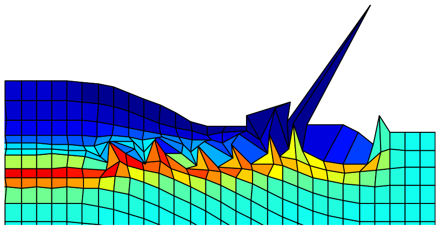
Moving Lagrangian Mesh

- high velocity impact

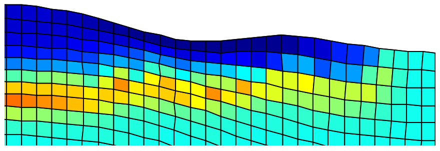
initial



Lagrangian



ALE



- computational mesh is fixed to the fluid and moves with the fluid
- moving mesh can degenerate
- degenerate typically for shear flow like high velocity impact, or vortex flow
- can be treated by ALE method

Euler Equations in Lagrangian Coordinates

- **density ρ, velocity $\mathbf{U}$, pressure p, internal energy $\epsilon = e - \mathbf{U}^2/2$, temperature T, heat conductivity κ, laser intensity I**

$$
\begin{aligned}
\frac{1}{\rho}\frac{\mathrm{d}\,\rho}{\mathrm{d}\,t} + \mathrm{div}\,\mathbf{U} &= 0, \qquad\qquad \frac{\mathrm{d}\,\mathbf{x}}{\mathrm{d}\,t} = \mathbf{U} \\
\rho\frac{\mathrm{d}\,\mathbf{U}}{\mathrm{d}\,t} + \mathbf{grad}\,p &= 0 \\
\rho\frac{\mathrm{d}\,\epsilon}{\mathrm{d}\,t} + p\,\mathrm{div}\,\mathbf{U} &= -\mathrm{div}(\mathbf{I}) + \mathrm{div}(\kappa\,\mathbf{grad}\,T)
\end{aligned}
$$

- **total Lagrangian time derivatives include convective terms**

$$
\frac{\mathrm{d}}{\mathrm{d}\,t} = \frac{\partial}{\partial\,t} + \mathbf{U}\cdot\mathbf{grad}
$$

- **equation of state – ideal gas and QEOS for plasma**
- **splitting – hyperbolic and parabolic part**
- **heat conductivity essential as it contributes to energy flux – faster shock waves**

ALE Method for Hydrodynamics

- **direct ALE – Arbitrary Lagrangian Eulerian method; Euler equations written in coordinates moving with speed $\mathbf{U_c}$ including convective terms (with factor $\mathbf{U - U_c}$; mesh movement is prescribed**

- **indirect ALE – combination of Lagrangian and Eulerian methods [Hirt, Amsden, Cook (JCP 1974, 1997)]**

  - **I. Lagrangian computation several time steps**
  - **II. Rezoning – mesh untangling and smoothing**
  - **III. Remapping – conservative interpolation of the conservative quantities from old to new, better quality mesh; then, back to Lagrangian computation.**

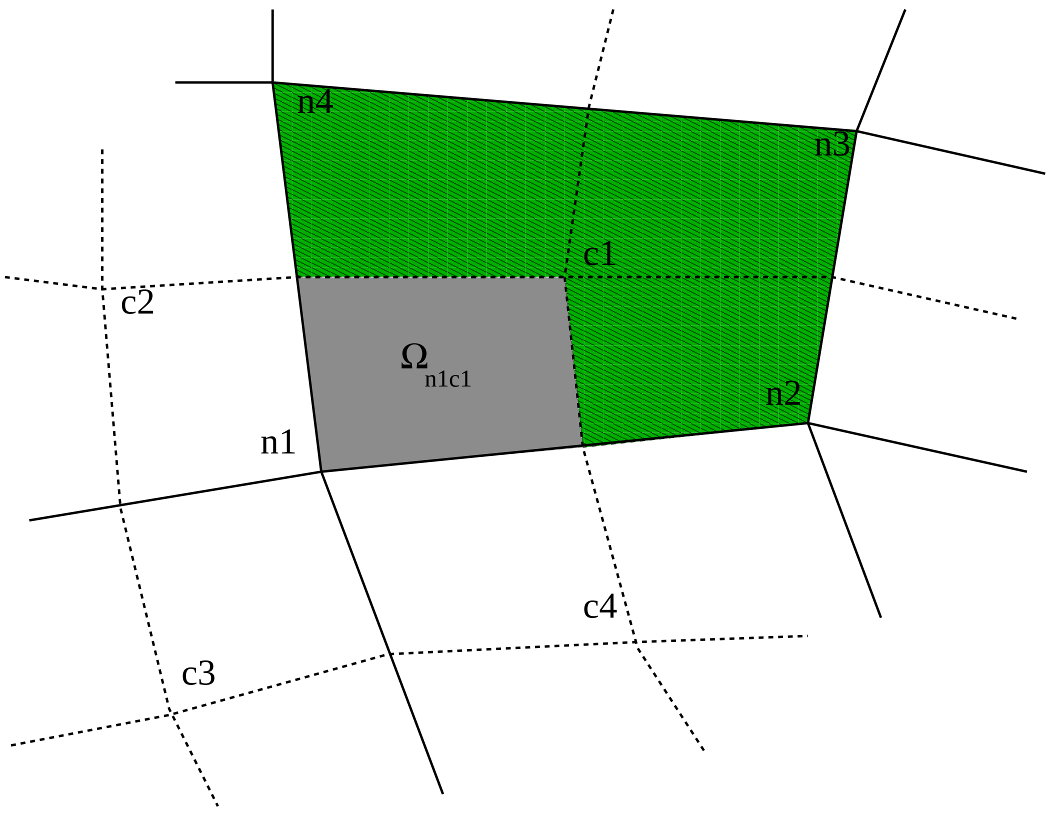
- **remapping (advection) corresponds to Eulerian part of ALE method, allows mass flux between cells**

- **ALE method combines positives of both approaches – grid moves with fluid (as Lagrangian), but Eulerian part keeps it smooth**

I. Lagrangian Step / Staggered Discretization

- **PALE is 2D code on quadrilateral, logically rectangular mesh**

- **cell (zone), node, sub-zone**
- **mass of sub-zone m_{nc}, mass of cell m_c, mass of node m_n**
- **staggered discretization – scalar quantities (density ρ, pressure p, internal energy ϵ, temperature T) defined in grid cells, vector quantities (positions $\mathbf{x}$, velocities $\mathbf{U}$) defined on grid nodes; density and pressure defined also in sub-zones**



I. Lagrangian Step / Energy Conservation

- **momentum equation**

$$m_n \frac{\mathrm{d}\mathbf{U}_\mathrm{n}}{\mathrm{dt}} = \mathbf{F}_n = \sum_{c \in \mathcal{C}(n)} \mathbf{F}_{cn}.$$

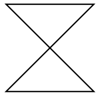
- **compatible formulation conserves total energy [Caramana, Burton, Shashkov, Whalen (JCP, 1998)]**

$$\begin{aligned}
\sum_c m_c e_c &= \sum_c m_c \epsilon_c + \sum_n \frac{1}{2} m_n (\mathbf{U}_n)^2, \\
\frac{\mathrm{d}}{\mathrm{dt}}\left(\sum_c m_c e_c\right) &= \sum_c m_c \frac{\mathrm{d}\epsilon_\mathrm{c}}{\mathrm{dt}} + \sum_n \underbrace{m_n \frac{\mathrm{d}\mathbf{U}_\mathrm{n}}{\mathrm{dt}}}_{=\mathbf{F}_n} \mathbf{U}_n, \\
&= \sum_c \left(m_c \frac{\mathrm{d}\epsilon_\mathrm{c}}{\mathrm{dt}} + \sum_{n \in \mathcal{N}(c)} \mathbf{F}_{cn} \cdot \mathbf{U}_n \right) = 0,
\end{aligned}$$

- **internal energy equation**

$$m_c \frac{\mathrm{d}\epsilon_\mathrm{c}}{\mathrm{dt}} = - \sum_{n \in \mathcal{N}(c)} \mathbf{F}_{cn} \cdot \mathbf{U}_n$$

I. Lagrangian Step / Forces



- **sub-zonal force $\mathbf{F}_{cn}$**

	pressure		artif. viscosity		anti-hourglass
$\mathbf{F}_{cn} =$	$\mathbf{F}_{cn}^{p}$	$+$	$\mathbf{F}_{cn}^{visco}$	$+$	$\mathbf{F}_{cn}^{\delta p}$

- **pressure force in sub-zone Ω_{cn} with boundary $\partial\Omega_{cn}$**

$$\mathbf{F}_{cn}^{p} = -\int_{\Omega_{cn}} \mathbf{grad}\, p \,\mathrm{d}V = -\int_{\partial\Omega_{cn}} p\, \mathbf{N} \,\mathrm{d}l.$$

- **artificial viscosity $q = c_1\rho_c a_c|\Delta\mathbf{U}| + c_2\rho_c(\Delta\mathbf{U})^2$, where $\Delta\mathbf{U} \approx \mathrm{div}\,\mathbf{U}\, l_c$ is velocity difference with l_c being characteristics length; added to pressure in compression regions; adds dissipation on shocks**
- **edge [Caramana, Shashkov, Whalen (JCP, 1998)] or tensor [Campbell, Shashkov (2000)] artificial viscosity**
- **sub-zonal pressure force prevents hourglass movement of cells – depends on difference between pressure in cell, and the pressure in sub-zones**
- **density in cell and sub-zone computed from mesh movement and Lagrangian assumption of constant sub-zonal mass**

II. Rezoning

- **rezoning – mesh untangling and smoothing**
- **for accurate remapping we need to move only those vertexes which are necessary and as little as possible; cell quality, node quality**
- **simple smoothing [Winslow (1963)]**

$$\begin{aligned}\mathbf{x}_{i,j}^{k+1} \quad = \quad & \frac{1}{2\left(\alpha^k+\gamma^k\right)}\Big(\alpha^k\left(\mathbf{x}_{i,j+1}^k+\mathbf{x}_{i,j-1}^k\right)+\gamma^k\left(\mathbf{x}_{i+1,j}^k+\mathbf{x}_{i-1,j}^k\right) \\ & -\frac{1}{2}\,\beta^k\left(\mathbf{x}_{i+1,j+1}^k-\mathbf{x}_{i-1,j+1}^k+\mathbf{x}_{i-1,j-1}^k-\mathbf{x}_{i+1,j-1}^k\right)\Big),\end{aligned}$$

where coefficients $\alpha^k = x_\xi^2 + y_\xi^2$, $\beta^k = x_\xi\, x_\eta + y_\xi\, y_\eta$, $\gamma^k = x_\eta^2 + y_\eta^2$, and where (ξ, η) are logical coordinates.

- **Reference Jacobian method [Knupp, Margolin, Shashkov (JCP, 2002)]**
- **combination of feasible set method and numerical optimization [Váchal, Garimella, Shashkov (JCP, 2004)].**

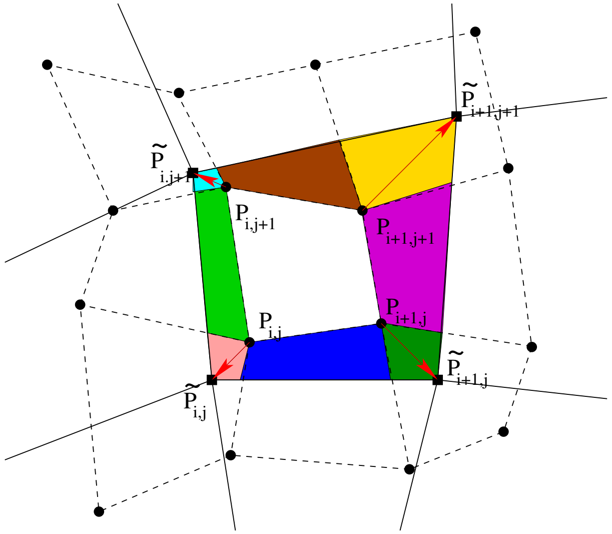
III. Remapping/1

- **conservative interpolation of conservative quantities from the old Lagrangian mesh to the new smoothed mesh**

  1. **piecewise linear reconstruction with Barth-Jespersen limiter [Barth, Jespersen (1989)]**

  $$g(x, y) = g_c + \left(\frac{\partial g}{\partial x}\right)_c (x - x_c) + \left(\frac{\partial g}{\partial y}\right)_c (y - y_c)$$

  2. **quadrature of reconstruction over cells of new mesh**
     - **exact quadrature – intersection of new cell with all neighboring old cells**

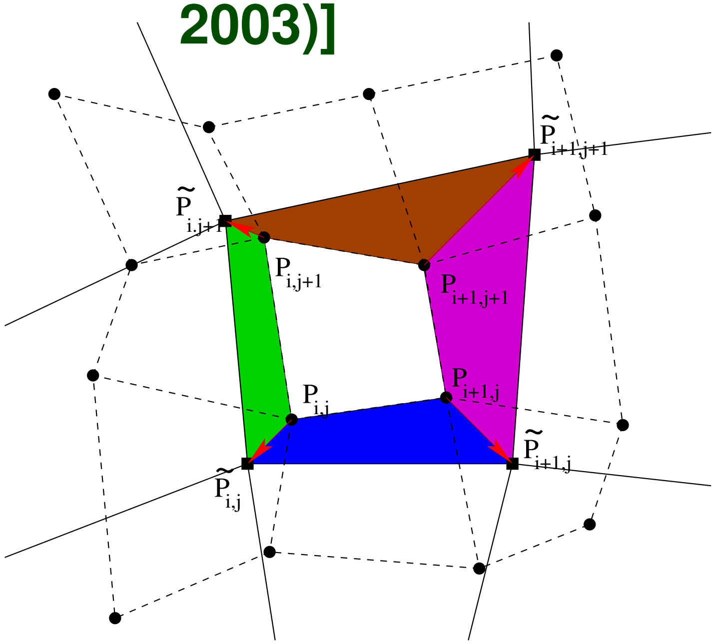


∗ **old mesh dashed, new mesh solid**

∗ **integration of linear function over each intersection polygon – Green theorem transforms into integration over polygon edges**

III. Remapping/2

– approximate quadrature over regions swept by edges moving form old to new position [Kuchařík, Shashkov, Wendroff (JCP, 2003)]



– exact integration is very expensive, requires finding intersections.

– integral over new cell can be decomposed as sum of integrals over swept regions.

3. repair – Barth-Jespersen limiter guarantees monotonicity in 1D; in 2D new local local extrema might appear – repair [Shashkov, Wendroff (JCP, 2004)]

- FCT remapping approach instead of repair, e.g. [Liska, Shashkov, Váchal, Wendroff (JCP, 2010)]
- remapping of staggered quantities more complicated [Loubere, Shashkov (JCP, 2005)]

Heat Conductivity / Formulation

- **heat conductivity represented as parabolic term in the energy equation; splitting parabolic part**

$$aT_t + \mathrm{div}\mathbf{w} = 0, \mathbf{w} = -\kappa\, \mathbf{grad}\, T = 0$$

- **mimetic operators method [Shashkov, Steinberg (JCP, 1996)]; operators**
  - **generalized gradient** $\mathbf{G}u = -\kappa\, \mathbf{grad}\, u$
  - **extended divergence** $\mathbf{D}\,\mathbf{w} = \begin{cases} \mathrm{div}\,\mathbf{w} & \textbf{on} \quad V \\ -(\mathbf{w}, \mathbf{n}) & \textbf{on} \quad \partial V \end{cases}$

- **divergence Green formula** $\displaystyle\int_V \mathrm{div}\,\mathbf{w}\, d\,V - \oint_{\partial V} (\mathbf{w}, \mathbf{n})\, d\,S = 0$

  is $(\mathbf{D}\,\mathbf{w}, 1)_H = 0$ **where** $(u, v)_H = \int_V u\, v\, d\,V + \oint_{\partial V} u\, v\, d\,S$

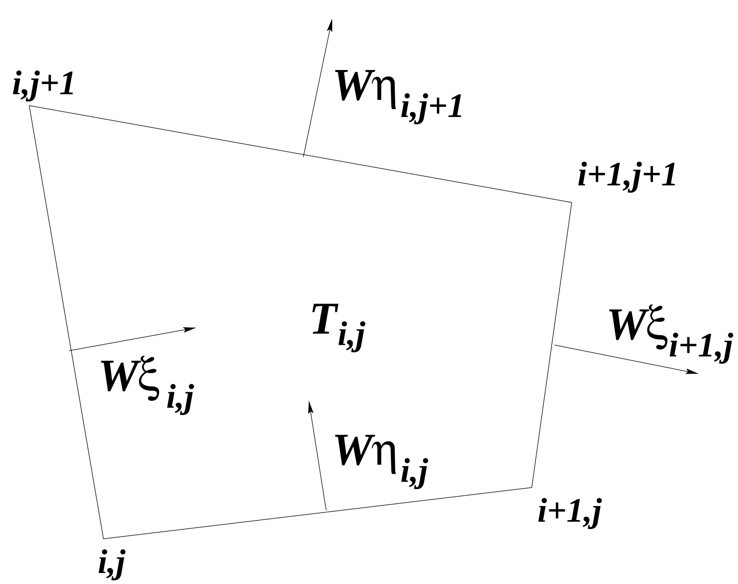
Heat Conductivity/ Divergence

- **divergence Green formula** $\int_V \operatorname{div}\mathbf{w}\, d\,V - \oint_{\partial V} (\mathbf{w},\mathbf{n})\, d\,S = 0$

  applied to one cell ij **gives standard discretization**

$$(\operatorname{div}\mathbf{W})_{ij} VC_{ij} = W\xi_{i+1,j} S\xi_{i+1,j} - W\xi_{ij} S\xi_{ij} + W\eta_{i,j+1} S\eta_{i,j+1} - W\eta_{ij} S\eta_{ij}$$

- **heat flux** $\mathbf{w}$ **represented at the center of each edge by the projections** $W\xi_{i,j}, W\eta_{i,j}$ **on normal to the edges**



Heat Conductivity / Gradient

- **Gauss theorem**

$$\int_V u \,\mathrm{div}\, \mathbf{w} \; d\,V - \oint u(\mathbf{w}, \mathbf{n}) \; d\,S + \int_V (\mathbf{w}, \kappa^{-1}\kappa \mathbf{grad}\, u) d\,V = 0$$

is $(\mathbf{D}\mathbf{w}, u)_H = (\mathbf{w}, \mathbf{G}u)_\mathbf{H}$ **where** $(\mathbf{A}, \mathbf{B})_\mathbf{H} = \int_V (\kappa^{-1}\mathbf{A}, \mathbf{B}) d\,V$

- $\mathbf{G}$ **is adjoin operator of** $\mathbf{D}$

$$\boxed{\mathbf{G} = \mathbf{D}^*}$$

- **mimetic discrete operators** G, D **have the same discrete integral properties**

- **namely** G **is constructed as adjoin of divergence** $G = D^*$ **from** D **using discrete inner products** $(u, v)_H, (\mathbf{A}, \mathbf{B})_\mathbf{H}$

- **gradient has a global stencil**

Heat Conductivity / System

- **implicit scheme in flux form**

$$a\frac{T^{n+1} - T^n}{\Delta t} + D\mathbf{W}^{n+1} = 0$$
$$\mathbf{W}^{n+1} - GT^{n+1} = 0$$

- **same time step as in hyperbolic Lagrangian/ALE step**
- **temperature T^{n+1} is eliminated and the system is solved for heat flux $\mathbf{W}^{n+1}$; linear operator with local stencil**
- **the sparse matrix of the system is symmetric and positive definite; solved by conjugate gradient method preconditioned by altered direction implicit (ADI) method; efficient solver**
- **having fluxes $\mathbf{W}^{n+1}$ temperature T^{n+1} given by**

$$T^{n+1} = T^n - \Delta t/aD\mathbf{W}^{n+1}$$

- **works well on bad quality Lagrangian meshes; allows discontinuous heat conductivity; non-linear substitution for non-linear (power) heat conductivity**

Heat Conductivity / Heat Flux Limiting

- **computed fluxes have to be smaller than physical heat flux limit** $|\mathbf{W}^{n+1}| < W_{limit}$
- **direct heat flux limiting** $\mathbf{W}^{n+1} = \mathrm{sign}\mathbf{W}^{n+1}\min(|\mathbf{W}^{n+1}|, W_{limit})$ **leads to temperature oscillations and checkerboard patterns**
- **in regions where physical heat flux limit is violated heat conductivity** κ **is replaced by**

$$\tilde{\kappa} = \kappa \frac{W_{limit}}{|\mathbf{W}^{n+1}|}$$

and limited heat fluxes are recomputed with new heat conductivity $\tilde{\kappa}$

Cylindrical Geometry

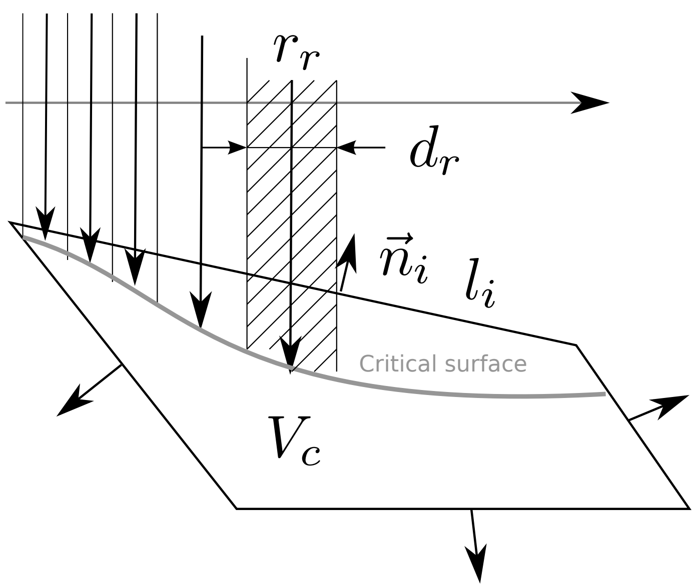
- generalized to cylindrical r, z geometry [Kuchařík, Liska, Loubere, Shashkov (HYP2006)] necessary for laser applications
- additional factor r in finite volumes integrals

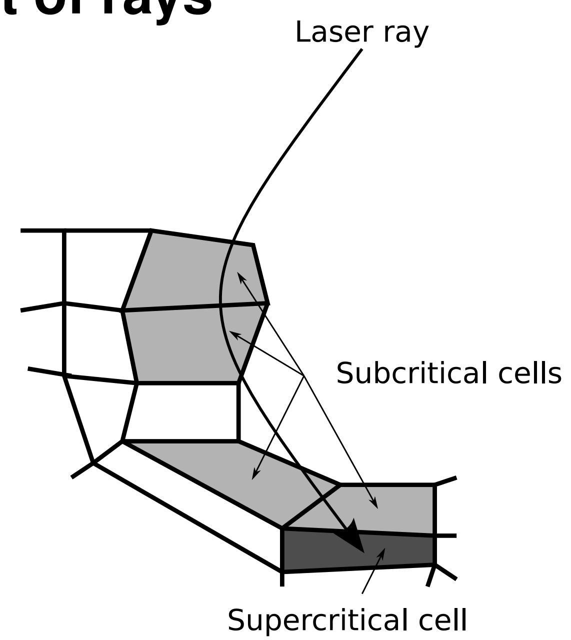
$$\int f(x,y)dxdy \rightarrow \int f(r,z)rdrdz$$

- Lagrangian step
  - control volume method
  - cell center moved to center of cell mass – so that ALE remapping can be conservative
- rezoning – mesh nodes move on the z axis
- remapping – additional factor r in integrals
- generalization of mimetic heat conductivity method to cylindrical geometry

Laser Absorption on Critical Surface

- **critical electron density** $n_e^c = \frac{m_e \pi c^2}{e^2 \lambda^2}$**; critical surface is the isosurface with** $n_e = n_e^c$
- **simplest model – laser penetrates till critical surface and is absorbed on the critical surface**
- **laser beam with parallel rays or Gaussian beam with angular divergence; laser beam split into set of rays**



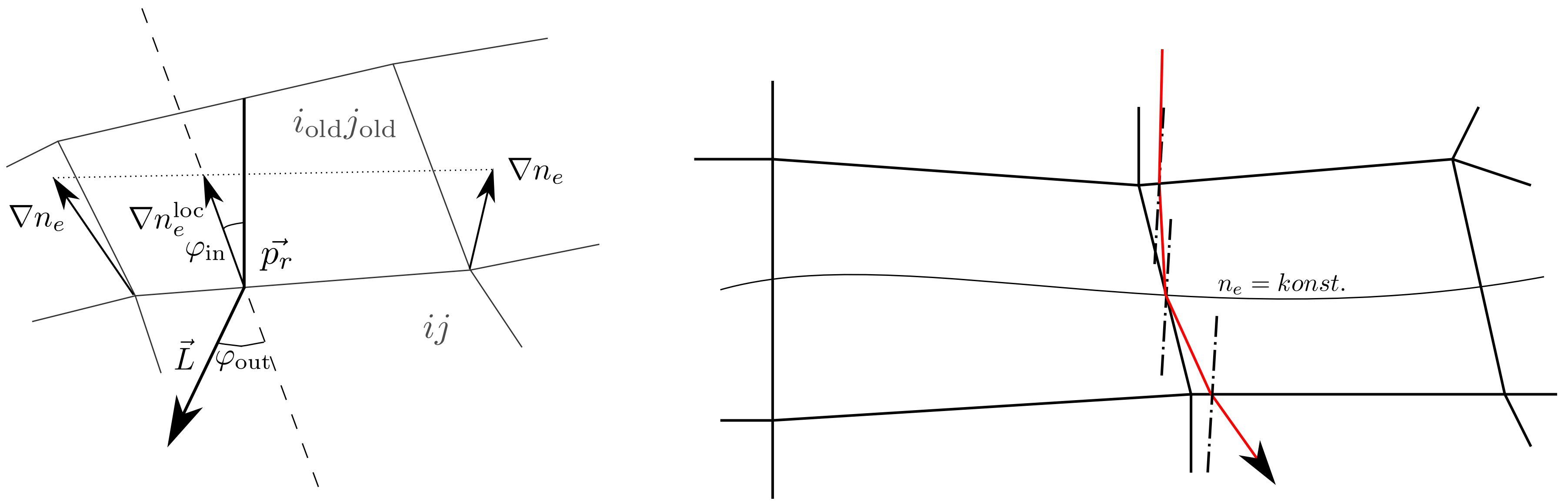


- **source in internal energy equation** $\rho \frac{d\,\epsilon}{d\,t} + p\,\mathrm{div}\,\mathbf{u} = -\mathrm{div}(\mathbf{I})$

Laser Absorption by Ray Tracing

- **laser beam split into rays; propagation of each ray through the computational mesh is simulated; rays are traced**

- **ray is refracted (Snell's law) when it passes through the edge from one cell to another; refraction line is orthogonal to ∇n_e**

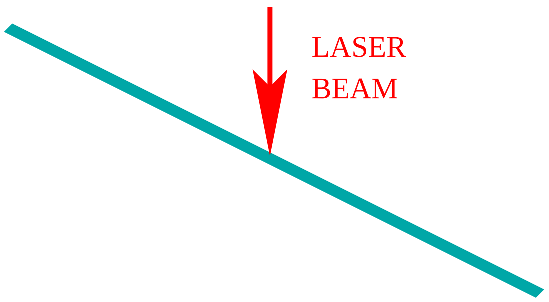


- **ray looses its energy by inverse bremsstrahlung by passing trough the cells**

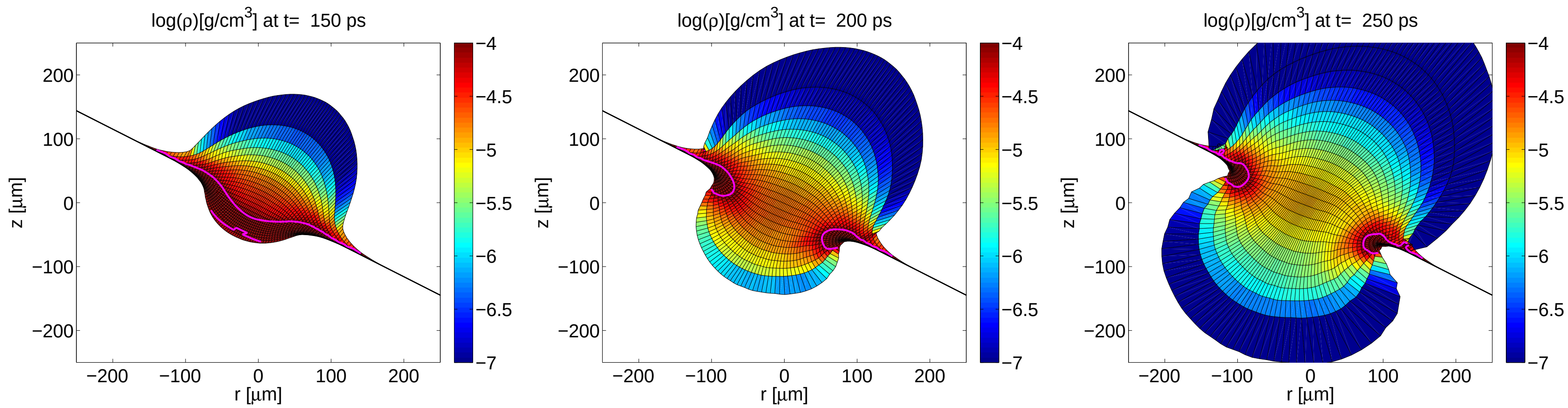
- **ray is gradually reflected close to the critical surface, where resonance absorption occurs**

Single Foil Target

- 30° **oblique incidence of laser on** $0.8\ \mu\mathrm{m}$ **thin Al foil; Cartesian geometry**



- **laser energy** $36\,\mathrm{J}$**, 3-rd harmonics, pulse length** $250\,\mathrm{ps}$**, focus** $r_f = 40\ \mu\mathrm{m}$



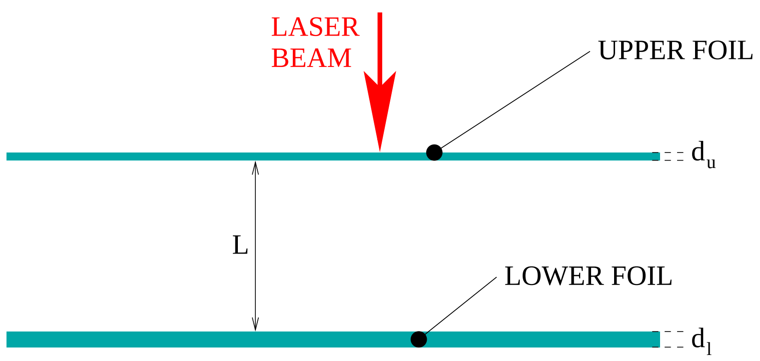
- **confirmed – plasma plumes propagate in direction orthogonal to the foil,** **animation**

  from pure Lagrangian simulation

- **preliminary study for double foil target; results for oblique and orthogonal laser incidence are very close**

Double Foil Target

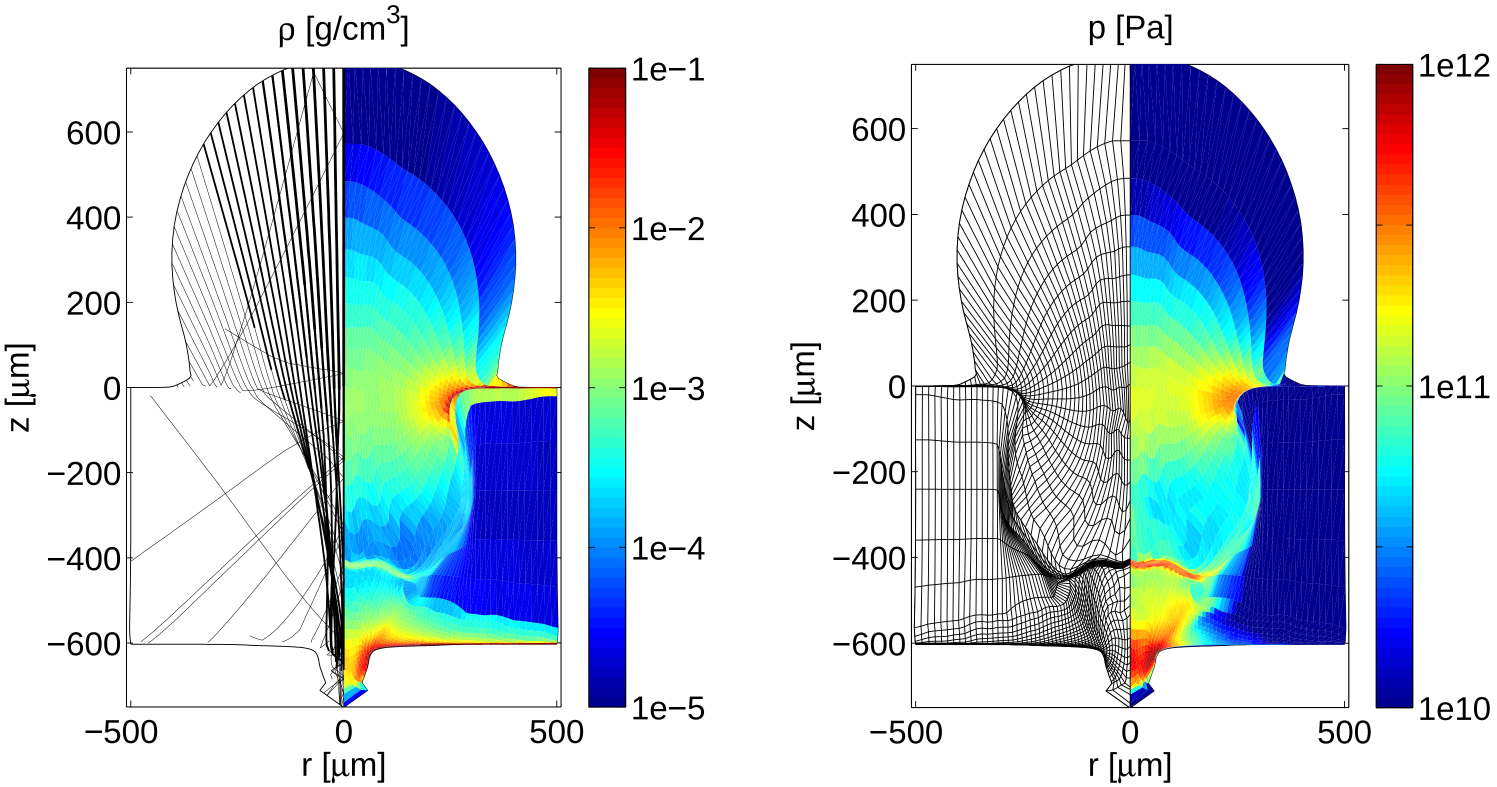
- upper Al and lower Mg foil
- foils thickness $d_u = 0.8\mu m, d_l = 2\mu m$
- foils distance $L = 600\mu m$
- Gaussian laser beam with energy $115\,\mathrm{J}$, 3-rd harmonics, pulse length $250\,\mathrm{ps}$, focus $r_f = 40\,\mu\mathrm{m}$, angular beam divergence $15°$, focused on the lower foil



- almost vacuum between foils; mass of neighboring vacuum and foils cells should not differ much; vacuum cells are big while foils cells small
- initially e.g. one foil rectangular cell has r/z edges lengths aspect ratio 10^4 and neighbors the vacuum cell with r/z ratio 0.2
- pure Lagrangian simulation fails due to mesh degeneration soon after laser burns through the upper foil

Double Foil Target Results

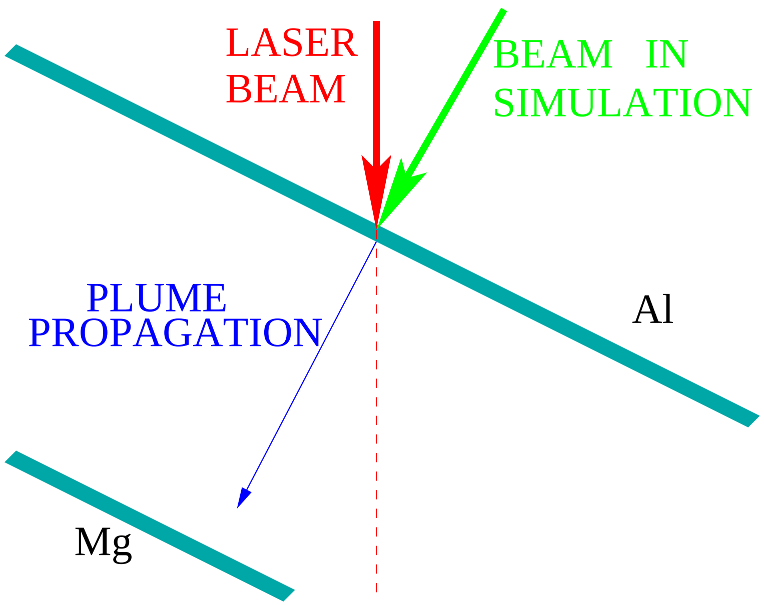
- **laser absorption by ray tracing**
- **density with selected rays, pressure with mesh at time 600 ps** **animation**



- **laser-produced plasma wall interaction [Renner, Liska, Rosmej (LPB 2009)]**

Oblique Incidence on Double Foil Target

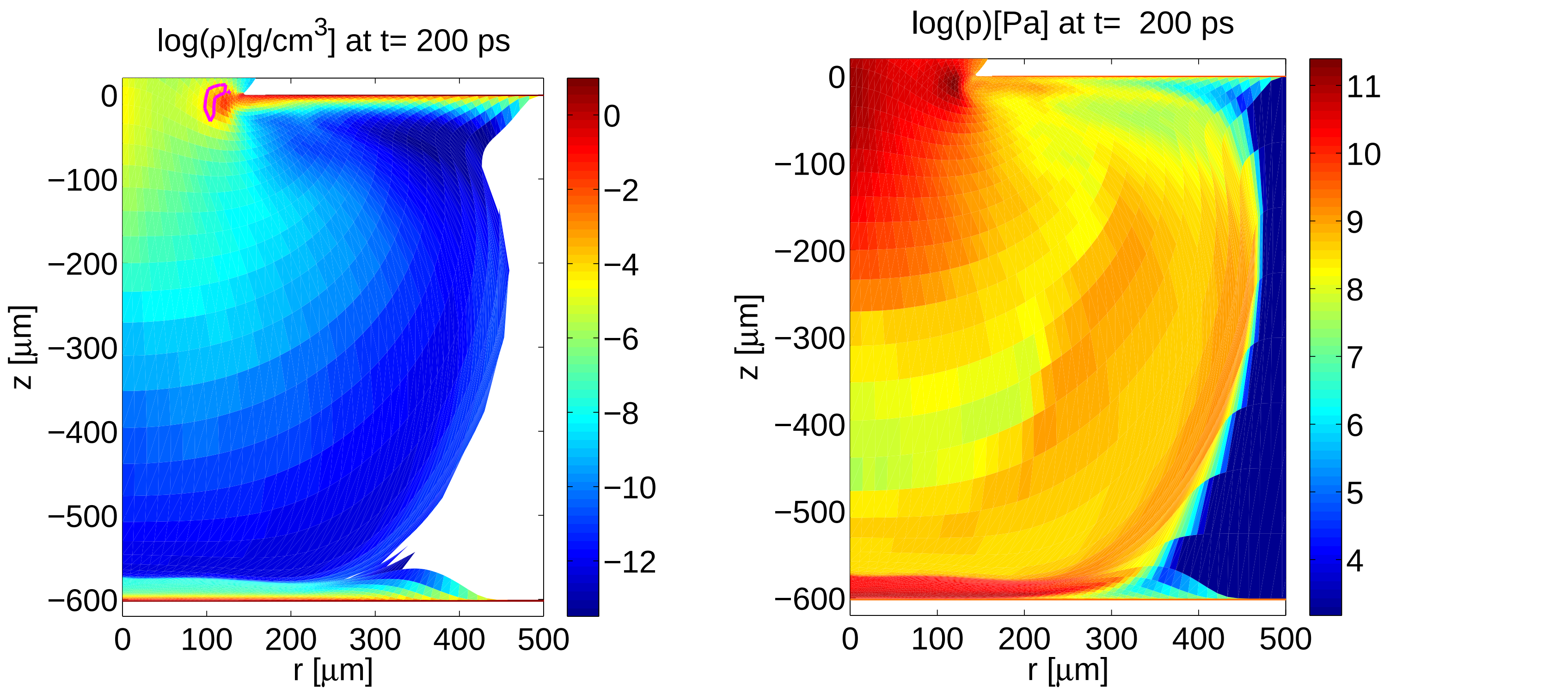
- **upper Al and lower Mg foil**
- **foils thickness** $d_u = 0.8\mu m, d_l = 2\mu m$**;** **foils distance** $L = 600\mu m$
- **Gaussian laser beam with energy** $100\,\mathrm{J}$**, 3-rd harmonics, pulse length** $250\,\mathrm{ps}$**, focus** $r_f = 40\,\mu\mathrm{m}$
- **after burning through the Al foil laser does not hit the Mg foil**



- **Al plasma plume propagates in direction orthogonal to foils**
- **oblique and orthogonal incidence produces very similar results**
- **simulation preformed in cylindrical geometry with symmetry axis being orthogonal to foils**
- **beam in simulation orthogonal to the foils; beam artificially stopped between the foils**

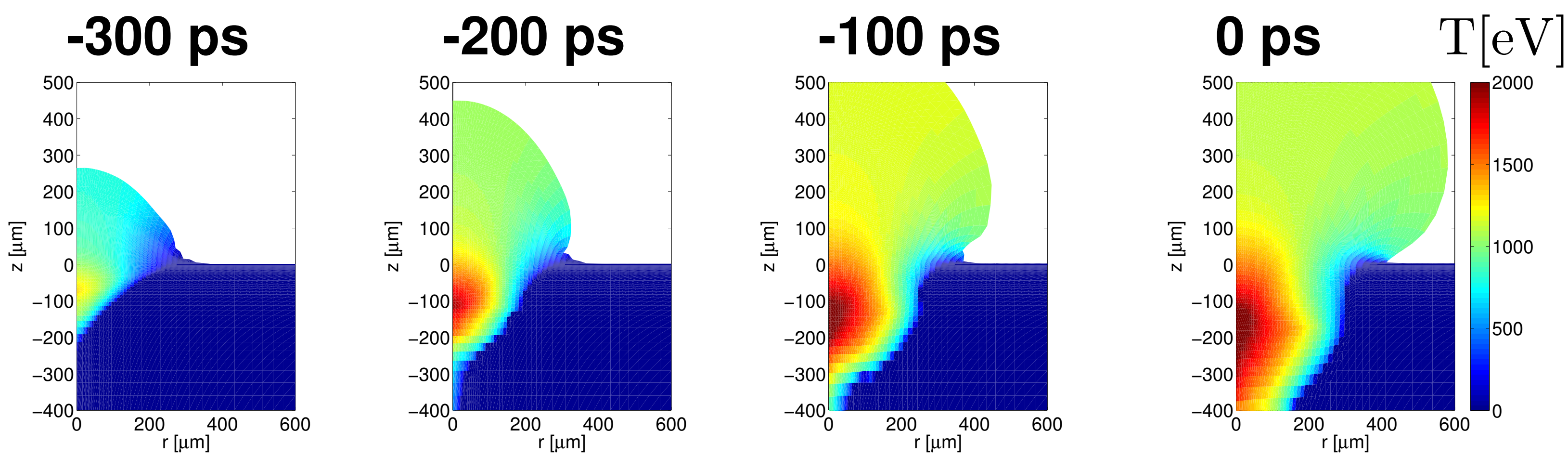
Oblique Incidence on Double Foil Target Results

- **3 materials, Aluminum, Magnesium and vacuum**
- **Mg foil heated by Al plasma plume; real plasma wall interaction**
- **density and pressure at time 500 ps** animation



Foam Target

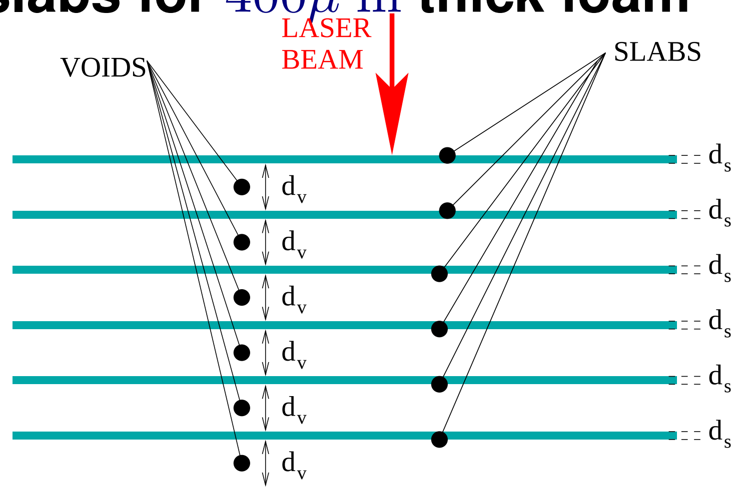
- 400μ m **thick TAC foam with density** $9.1\text{mg}/\text{cm}^3$ **with** $2\mu\text{m}$ **pores**
- **Gaussian laser pulse on the third harmonics with wavelength** $0.438\,\mu\text{m}$**, total energy** $170\,\text{J}$**, the radius of laser spot on target** $300\,\mu\text{m}$ **and FWHM length** $320\,\text{ps}$
- **foam modeled by uniform density** $9.1\text{mg}/\text{cm}^3$ **material**



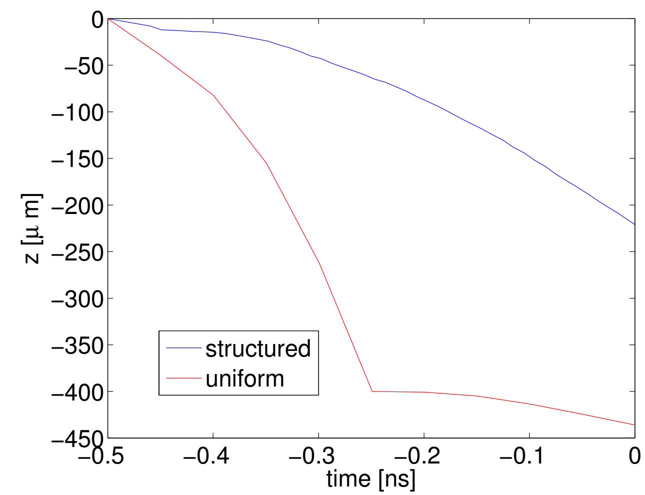
evolution of temperature; timing relates to the laser pulse maximum at $0\ \text{ps}$

Foam Target - Structured Model

- **foam modeled by the sequence of $d_s = 0.018\mu\text{m}$ thick dense slabs with density $\rho_s = 1\ \text{g/cm}^3$ separated by $d_v = 1.982\mu\text{m}$ thick voids with density $\rho_v = 1\ \text{mg/cm}^3$**
- **thickness of a slab and void is $d_s + d_v = 2\mu\text{m}$, i.e. we have 200 slabs for 400μ m thick foam**



structured foam model

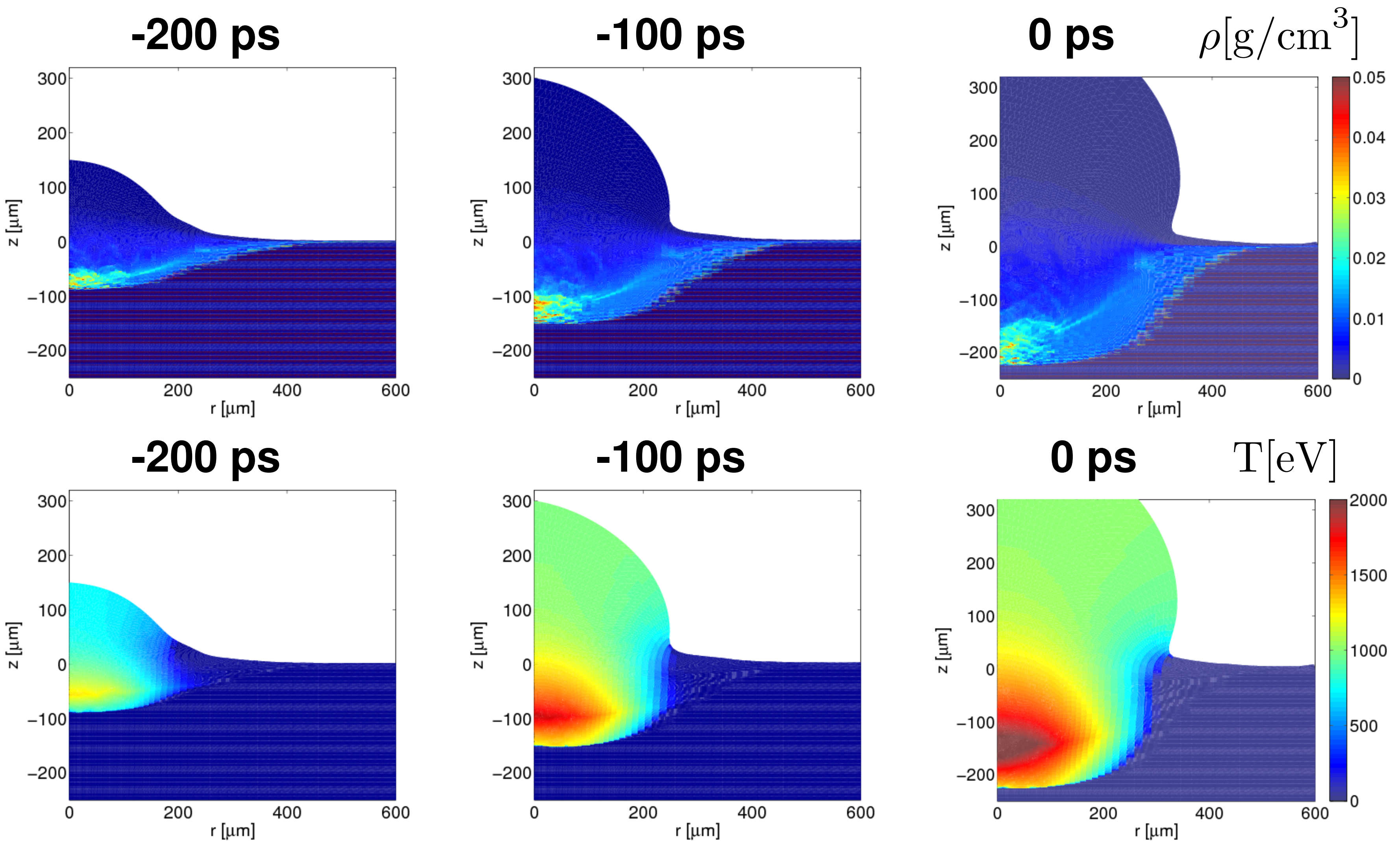


burning of laser through the target

- **experimental speed of laser penetration into the foam is about $600 \sim 700\ \mu\text{m/ns}$, speed from structured simulation is about $500\ \mu\text{m/ns}$ and from uniform simulation about $1600\ \mu\text{m/ns}$**
- **structured model approximates experimental data much better [Kapin, Kuchařík, Limpouch, Liska, (2006)]**

Foam Target - Structured Model Results

- evolution of density and temperature

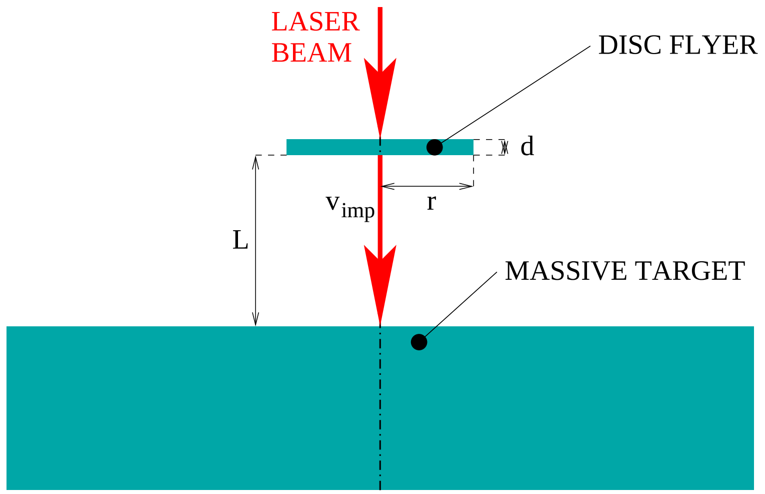


- density animation

 , zoomed animation

High Velocity Impact

- **disc flyer impact problem**
- **high power laser-irradiated Aluminum disc ablatively accelerates up to very high velocity (40-190 km/s) and strikes to massive Aluminum target**
- $d = 6; 11\mu m, r = 150\mu m, L = 200\mu m$, **laser energy** $120 - 390\,\mathrm{J}$, **1-st or 3-rd harmonics, pulse length** $400\,\mathrm{ps}$, **focus** $r_f = 125\,\mu\mathrm{m}$.

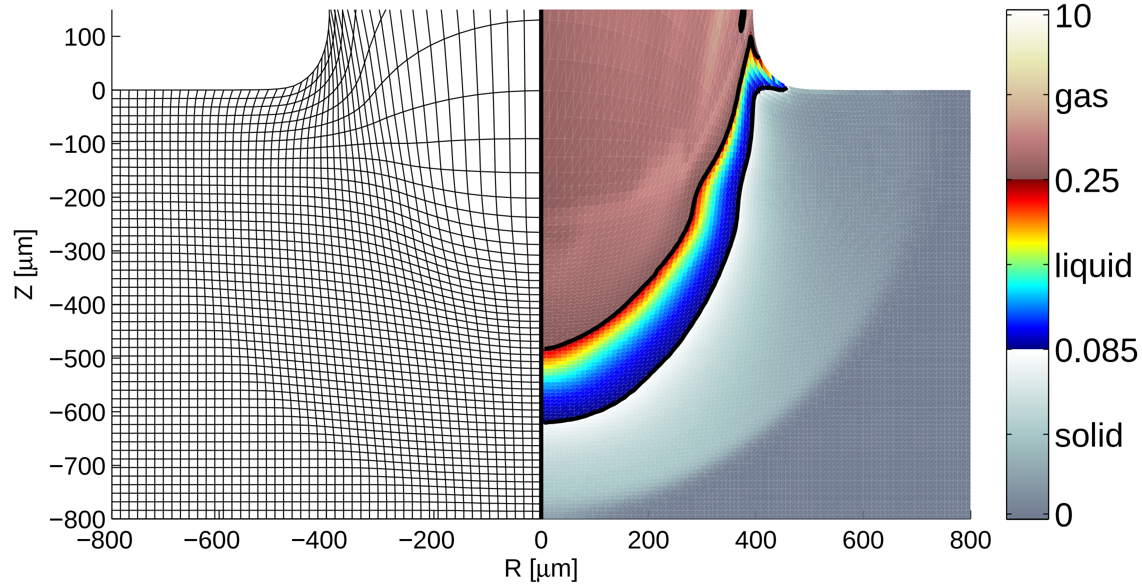


- **problem split into two parts for simulations:**
  - **ablative disc flyer acceleration by laser beam; animation**
  - **impact of disc flyer into massive target**
- **problem parameters similar to the experiment performed on the PALS laser facility in Prague**

Crater Creation

- **after impact – increase of temperature, melting and evaporating material, circular shock wave**

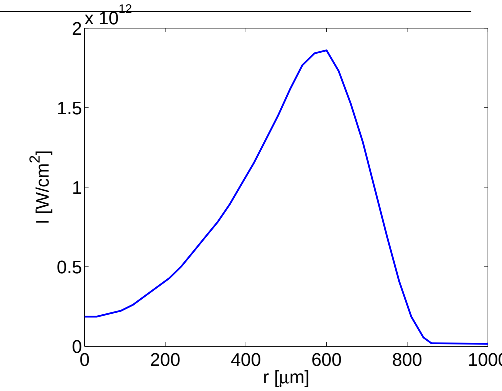
- **crater (gas - liquid interface) formed inside the target**



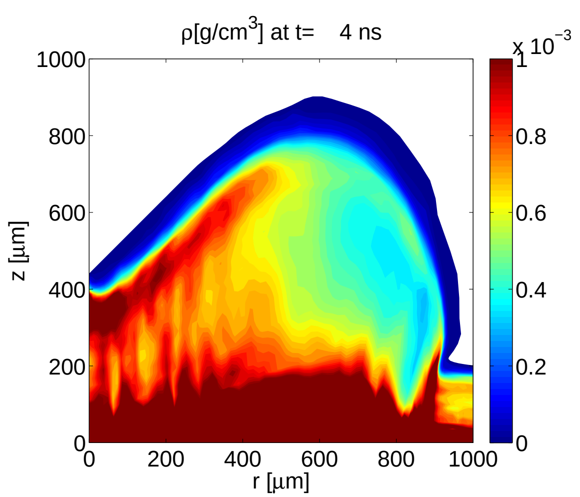
- **temperature** **animation**

- **simulated craters size and shape correspond reasonably well to experimental data [Kuchařík, Liska, Limpouch (2006)]**

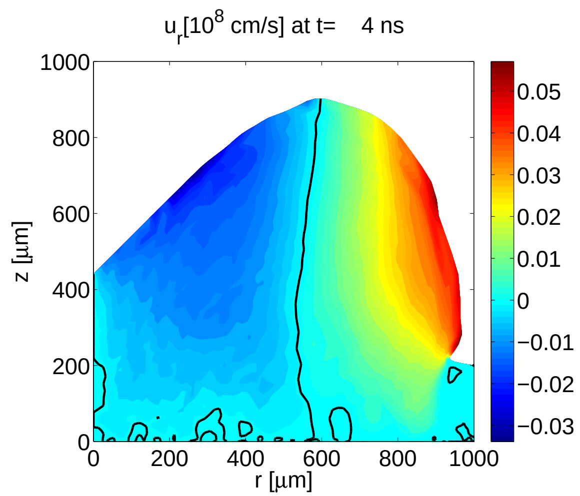
Jets Formation



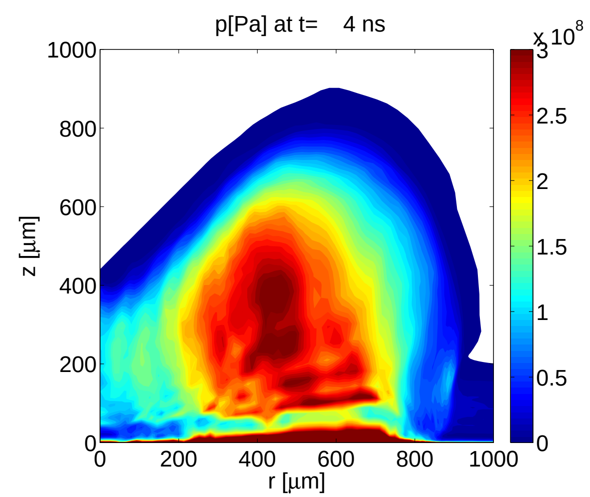
- laser on 3-rd harmonics, total energy $10\mathrm{J}$, FWHM $400\mathrm{ps}$, heat flux limiter 5 %
- annular laser profile having 10% at $r = 0$, smooth maximum at $r = 600\mu m$ and proportional to r^2 for small r
- plasma plume develops faster on circle of laser maximum
- inner part of plume moves inwards towards z axis; pressure gradient towards z axis



density



radial velocity

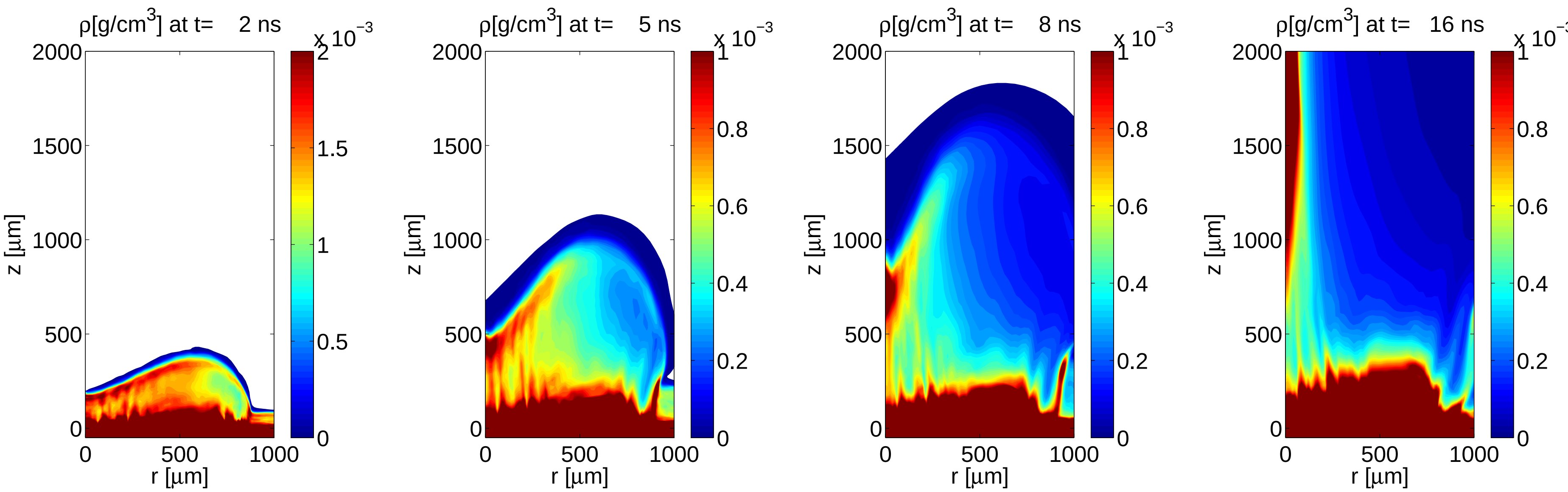


pressure

- conical profile in density collides on the z axis creating a jet

Jets Evolution

- **density evolution, animation**



- **pure hydrodynamics process of jet formation from annular laser profile [Kmetík, Limpouch, Liska, Váchal (2011)]**

- **role of other physical processes as radiation transport**

Conclusion

- ALE method for hydrodynamics in Cartesian and cylindrical geometry using staggered Lagrangian scheme
- heat conductivity, laser absorption
- applications – simulations of single foil, double foil, foam, disc flyer targets and jets formation
- often pure Lagrangian simulation fails while ALE gives reasonable results
- simulations serve for interpretation of experimental results obtained on PALS laser facility