EL5823/BE6203 --- Medical Imaging - I

MRI Image Reconstruction and Image Quality

Yao Wang
Polytechnic University, Brooklyn, NY 11201

Based on J. L. Prince and J. M. Links, Medical Imaging Signals and Systems, and lecture notes by Prince. Figures are from the textbook except otherwise noted.

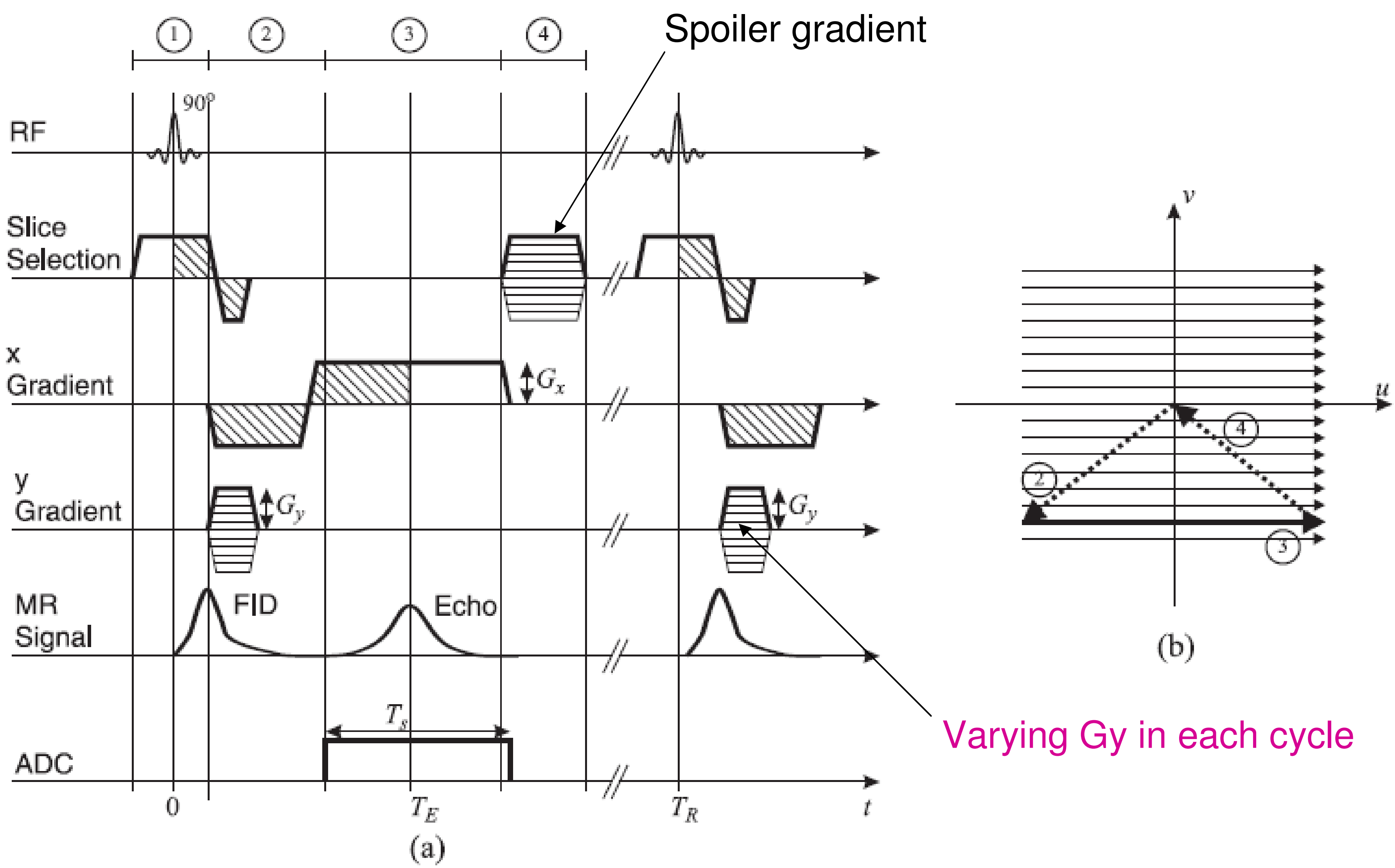
Lecture Outline

- Review of relation between pulse sequences and scanning trajectory
- Image reconstruction
  - Rectilinear scan
  - Polar scan
- Image quality
  - Sampling interval in Fourier space vs. field of view
  - Coverage area vs. blurring
  - Noise and SNR

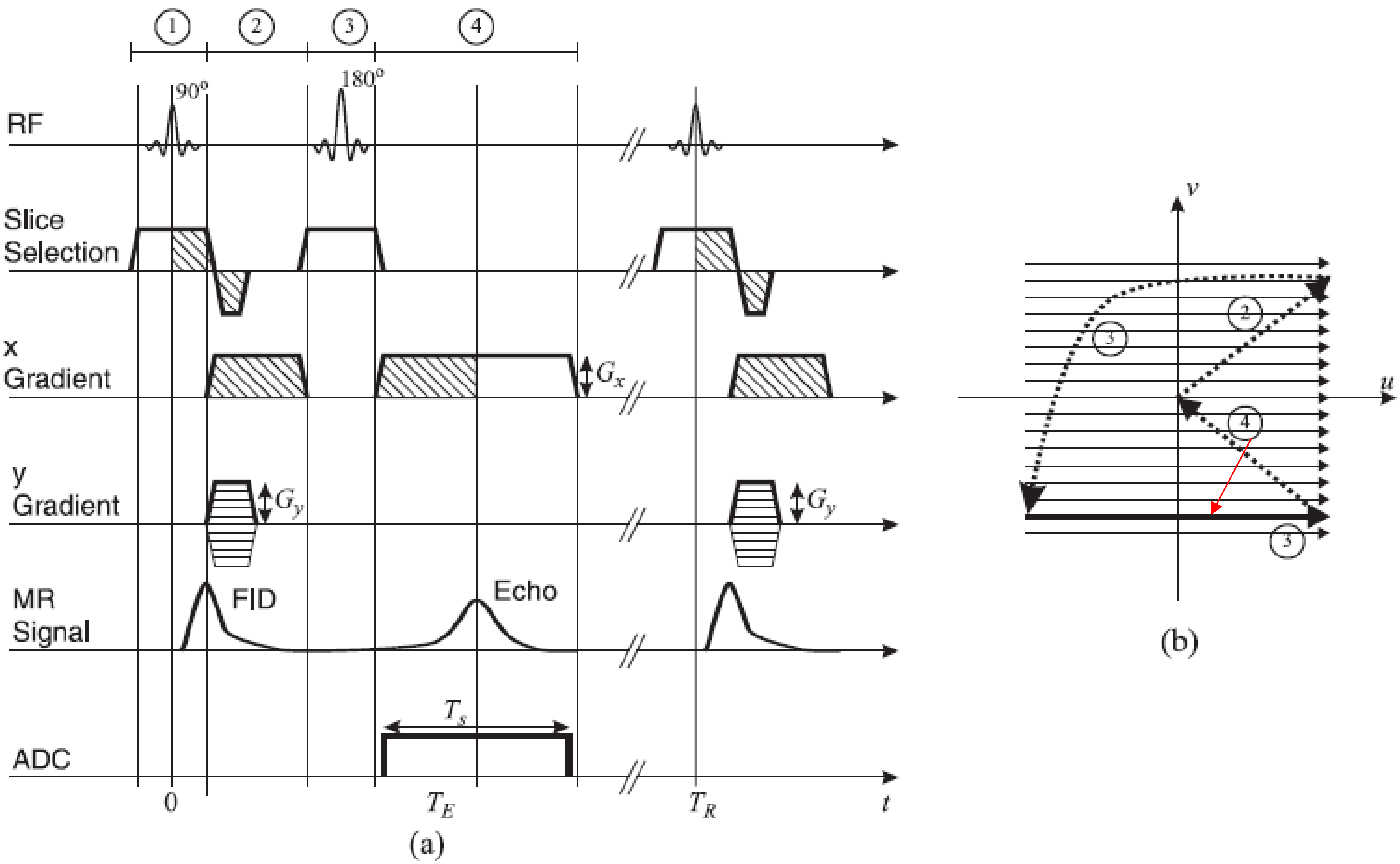
MRI Scan Review

- How to measure the signal at one particular location?
  - Using Z-gradient to vary the static field at different slices
  - Using RF pulses with a certain freq. to excite one slice at a time
  - Using X-gradient and Y-gradient to differentiate voxels in a slice
  - Polar scan
    - Apply X- and Y-gradient simultaneously with a given ratio, to scan one polar line
  - Rectilinear scan
    - Apply Y-gradient first to select one horizontal line in Freq. space
    - Apply X-gradient to scan the line
- Received signal is samples of the 2D Fourier transform over a slice
- How to obtain the original signal?

Realistic Gradient Echo Pulse Sequence



Realistic Spin Echo Pulse Sequence



Realistic Spin-Echo Polar Pulse Sequence

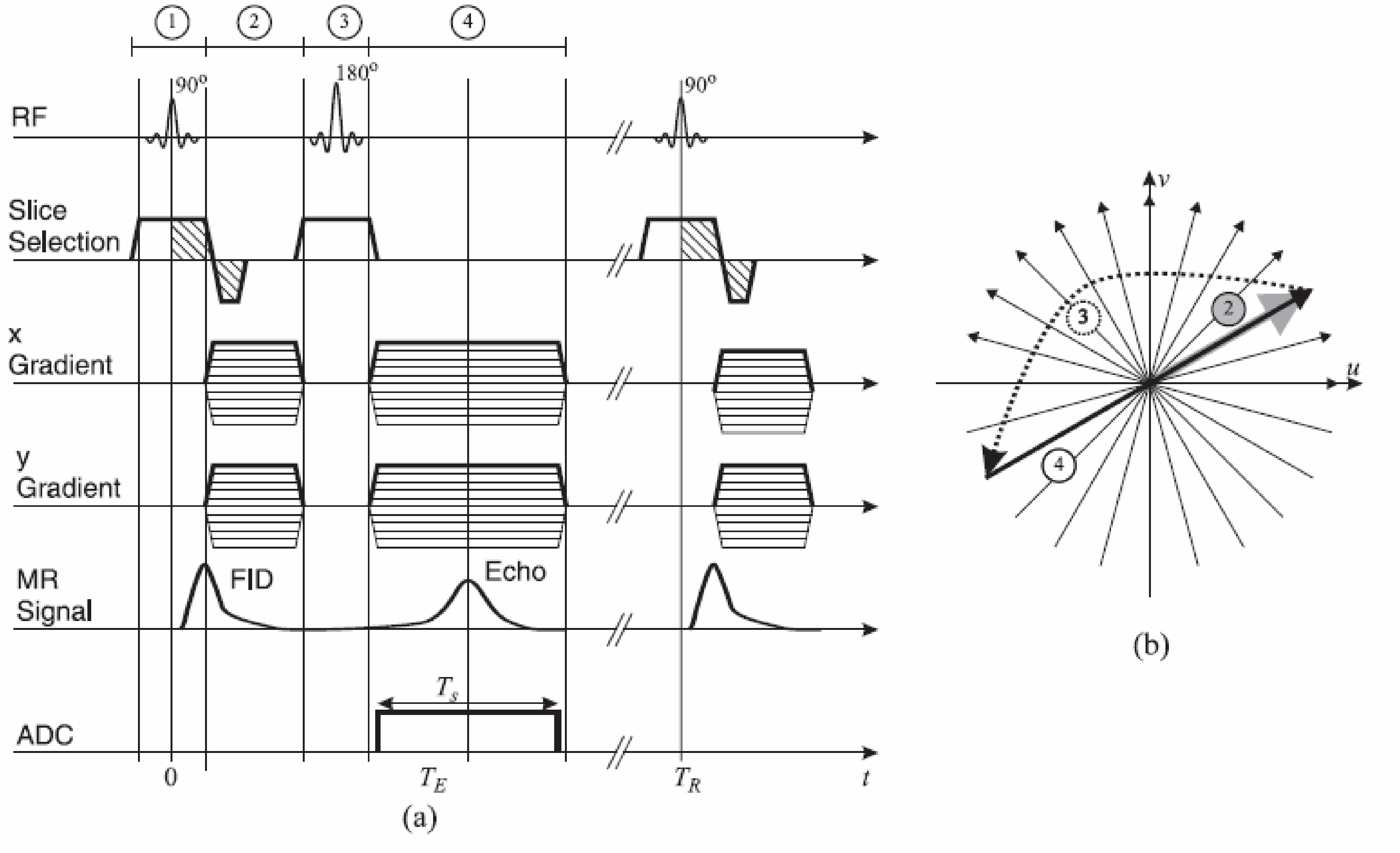


Image Reconstruction

- Rectilinear scan
  - Acquired signal is the samples of F(u,v) on a rectangular grid
  - Use inverse 2D FT
- Polar scan
  - Acquired signal is the samples of F(u,v) on the polar grid
  - Use inverse 2D FT after interpolation to rectangular grid
  - Or apply backprojection approach

Acquired Rectilinear Data

- Acquire data for all phase encode areas

$$A_y = G_y T_p$$

- Baseband signal

$$s_0(t, A_y) = \iint f(x, y) e^{-j\gamma G_x x t} e^{-j\gamma A_y y} dx dy$$

- Identify Fourier frequencies

$$\begin{aligned} u &= \gamma G_x t \\ v &= \gamma A_y \end{aligned}$$

Reconstruction from Rectilinear Scan

- Fourier transform is built over repetitions

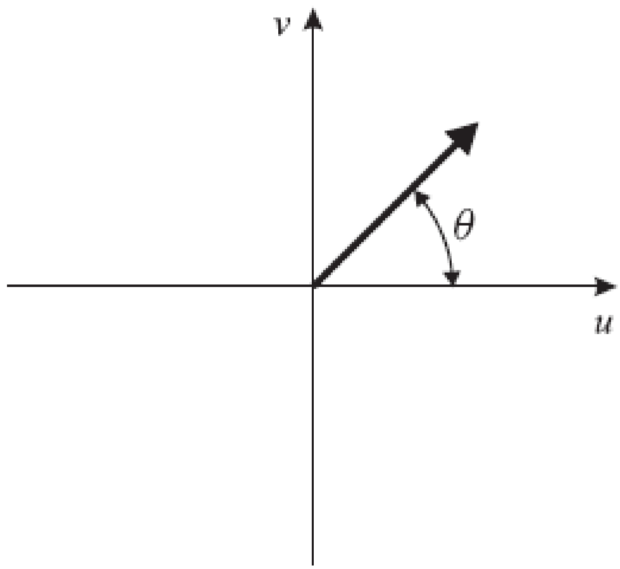
$$F(u,v) = s_0\left(\frac{u}{\gamma G_x}, \frac{v}{\gamma}\right) \quad 0 \le u \le \gamma G_x T_s$$

- Inverse Fourier transform

$$f(x,y) = \iint s_0\left(\frac{u}{\gamma G_x}, \frac{v}{\gamma}\right) e^{+j2\pi(ux+vy)} dx dy$$

- This is a fundamental equation in MRI

Acquired Polar Data



$$u = \gamma G_x t, v = \gamma G_y t$$

$$\rho = \gamma t \sqrt{G_x^2 + G_y^2}$$

$$\theta = \tan^{-1} \frac{G_y}{G_x}$$

$$s_0(t; G_x, G_y) = F(\rho \cos\theta, \rho \sin\theta) = G(\rho, \theta)$$

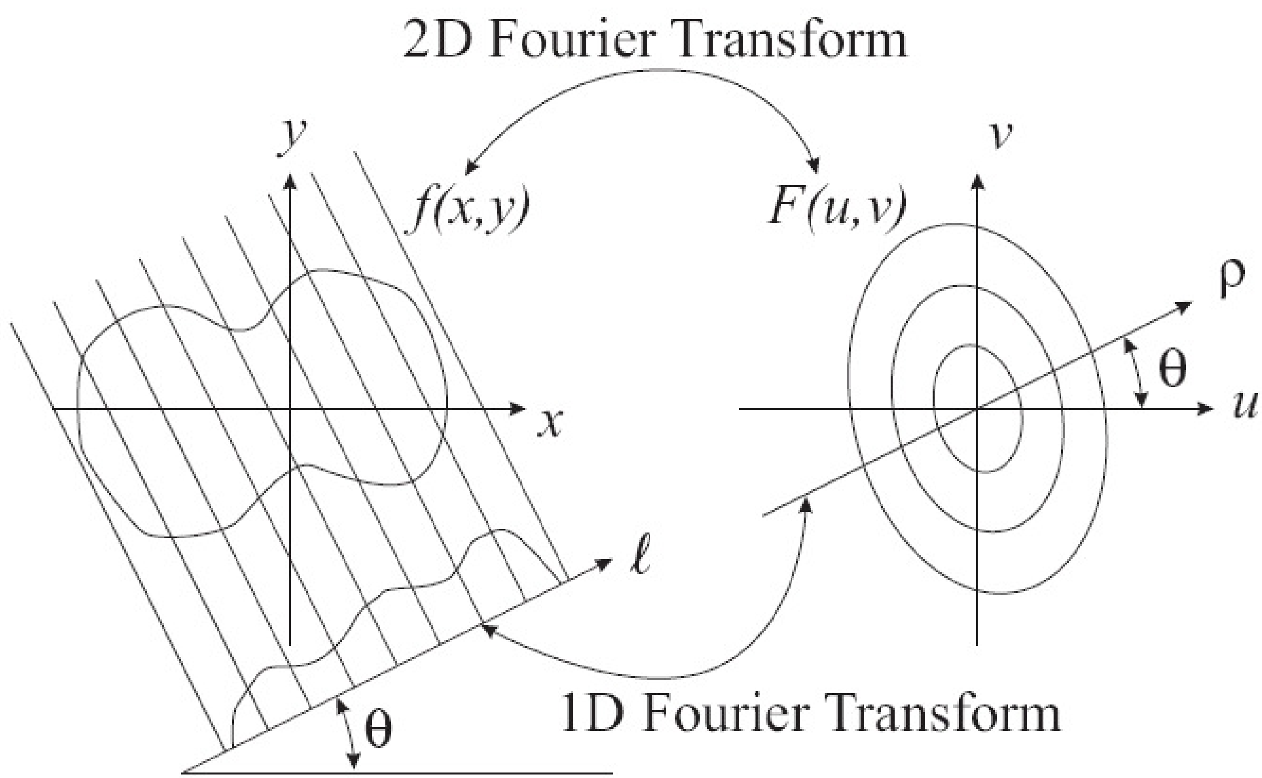
$$G(\rho, \theta) = s_0\left(\frac{\rho}{\gamma \sqrt{G_x^2 + G_y^2}}; G_x, G_y\right)$$

In each RF pulse cycle, a different Gx,Gy is used to form a different \theta
Within each cycle, during the ADC read out time, a range of \rho is achieved

Review: Projection Slice Theorem

- Projection Slice theorem
  - The Fourier Transform of a projection at angle θ is a line in the Fourier transform of the image at the same angle.

$$G(\varrho, \theta) = F(\varrho \cos\theta, \varrho \sin\theta)$$



Review: Reconstruction Algorithm for Parallel Projections

- Backprojection:
  - Backprojection of each projection
  - Sum

$$f_b(x, y) = \int_0^{\pi} [g(\ell, \theta)]_{\ell = x\cos\theta + y\sin\theta}\, d\theta$$

- Filtered backprojection:
  - FT of each projection
  - Filtering each projection in frequency domain
  - Inverse FT
  - Backprojection
  - Sum

$$f(x, y) = \int_0^{\pi} \left[\int_{-\infty}^{\infty} |\varrho| G(\varrho, \theta) e^{+j2\pi\varrho\ell} d\varrho\right]_{\ell = x\cos\theta + y\sin\theta} d\theta$$

- Convolution backprojection
  - Convolve each projection with the ramp filter
  - Backprojection
  - Sum

$$f(x, y) = \int_0^{\pi} [c(\ell) * g(\ell, \theta)]_{\ell = x\cos\theta + y\sin\theta}\, d\theta$$

Reconstruction from Polar data

- Method 1: filtered backprojection
  - In MRI, we measure G(\rho,\theta) directly. No transform of g(l,\theta) needed!

$$f(x,y) = \int_0^{\pi} \left[\int_{-\infty}^{\infty} |\varrho| G(\varrho,\theta) e^{+j2\pi\varrho\ell} d\varrho \right]_{\ell = x\cos\theta + y\sin\theta} d\theta$$

- Method 2: convolution backprojection
  - Must apply inverse 1D FT to G(\rho,\theta) to yield g(l,\theta)
  - Not as efficient
- Method 3:
  - Convert G(\rho,\theta) to rectangular grid F(u,v)
  - Apply inverse 2D FT
  - Not advisable

Image Quality

- Sampling parameters in Fourier space
  - Sampling spacing vs. field of view
  - Coverage area vs. blurring
- SNR

Nyquist Sampling Theorem: Review

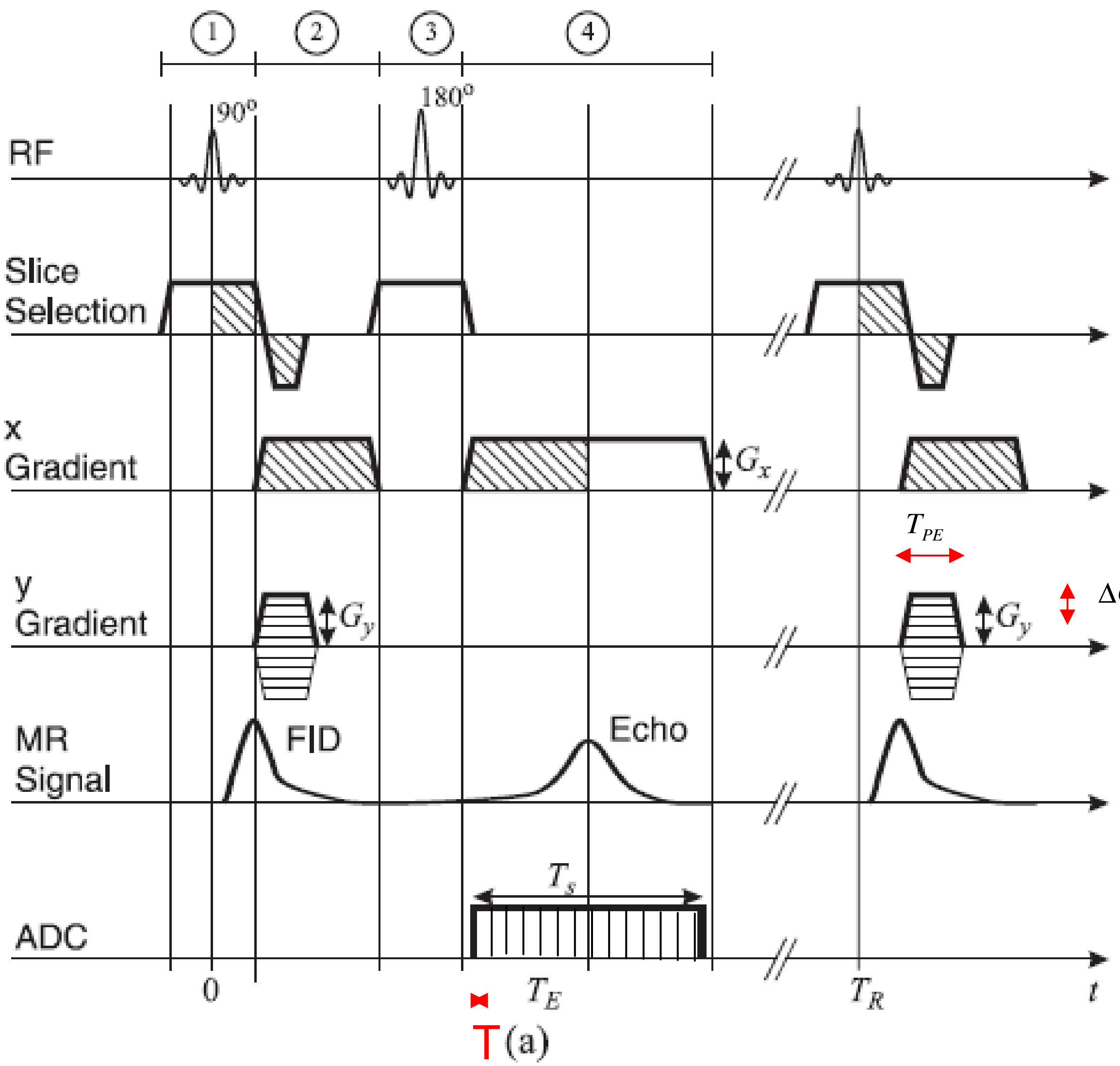
- Continuous signal with maximum freq f_{max}
- Must sample at $f_s \geq 2f_{max}$ (or $T \leq 1/2f_{max}$) to avoid aliasing
- When sampled at lower freq., high freq. wrap around low freq. (aliasing)

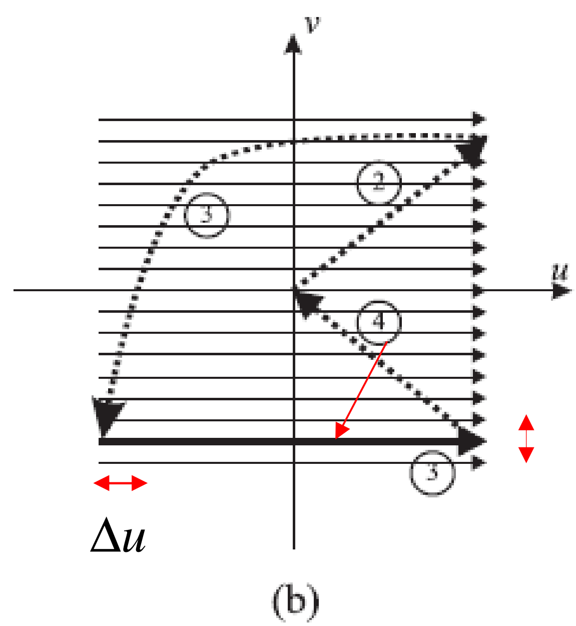
- Sampled signal with sampling interval T
- Maximum freq. $f_{max} = \frac{1}{2} f_s = 1/2T$

Sampling in MRI

- Slice selection: sampling in z-direction
  - Slice thickness \delta z controlled by RF excitation bandwidth \delta v
  - To avoid aliasing:
    - 1/\delta z >= 2 f_max,z -> \delta z <= 1/ (2 f_max,z)
- Within each slice, we sample in the Fourier domain (u,v)
  - (called k-space in MRI literature, kx=u, ky=v)
  - Rectilinear Scan
    - \delta u depends on sampling interval T during readout (ADC)
    - \delta v depends on spacing between phase encoding
  - Polar scan
    - Angle spacing depends on steps in Gy/Gx
    - \rho spacing: depends on sampling interval T during readout
  - We will discuss rectilinear scan only

Rectilinear Scan





$$\Delta u = \gamma G_x T$$

$$\Delta v = \gamma \Delta G_y T_{pe}$$

Sampling in u

- Recall each pulse sequence contains an ADC window
  - Data are acquired by a A/D converter during this time
  - N samples are taken during Ts
  - Sampling interval T=Ts/N
  - Sampling rate fs=1/T=N/Ts
  - Sampling step in u $$\Delta u = \gamma G_x T$$
- The signal is demodulated and then sampled
- ADC uses an antialiasing filter with support region (-fs/2, fs/2), bandwidth = fs (receiver bandwidth)
- X-gradient relates x with Larmor freq v by
  - v = v0+ \gamma Gx x
- Only signals with freq = v0+/- fs/2 are measured
  - Correspond to x_min=x0-fs/2/\gamma Gx, x_max= x0 + fs/2/\gamma Gx
  - Field of view FOV_x = x_max-x_min=fs/\gamma Gx = 1/\gamma Gx T

$$\mathrm{FOV}_x = \frac{f_s}{\gamma G_x} = \frac{1}{\gamma G_x T}$$

- Smaller T -> Large FOV_x

Sampling in V

- Phase encoding gradient Gy, phase =\gamma Gy T_PE
- Each time change G_y by \Delta G_y, or A=Gy T_PE by \delta A_y
- Step in v

$$\Delta v = \gamma \, \Delta A_y$$

- Field of view in y:

$$\mathrm{FOV}_y = \frac{1}{\gamma \, \Delta A_y} = \frac{1}{\Delta v}$$

- No explicit anti aliasing filt
- Lack of antialiasing filter could cause wrap around
  - Axial slice of brain: front appear in back
  - Smaller \Delta A_y -> large FOV_y
- We often choose \Delta A_y so that \delta v = \delta u (or \Delta A_y=GxT, or \Delta G_y = G_x T/T_pe)

Resolution of MRI

- MRI scan covers only a finite area of the Fourier space
- Actual Fourier transform may be non-zero outside this

  - Fourier space coverage

$$U = N_x \gamma G_x T$$
$$V = N_y \gamma \Delta A_y$$

  - Implied lowpass filter is

$$H(u,v) = \text{rect}\left(\frac{u}{U}\right)\text{rect}\left(\frac{v}{V}\right)$$

  - Spatial PSF is

$$h(x,y) = UV\text{sinc}(Ux)\text{sinc}(Vy)$$

Reconstructed signal: \hat f(x,y)= f(x,y) * h(x,y)

Width of the Blurring Function

- Effective width of sinc function = main lobe/2 = first zero

- FWHMs are

$$\mathrm{FWHM}_x = \frac{1}{U} = \frac{1}{N_x \gamma G_x T} = \frac{1}{N_x \Delta u}$$
$$\mathrm{FWHM}_y = \frac{1}{V} = \frac{1}{N_y \gamma \Delta A_y} = \frac{1}{N_y \Delta v}$$

Increasing U, V (coverage area in Fourier space) reduces blurring!

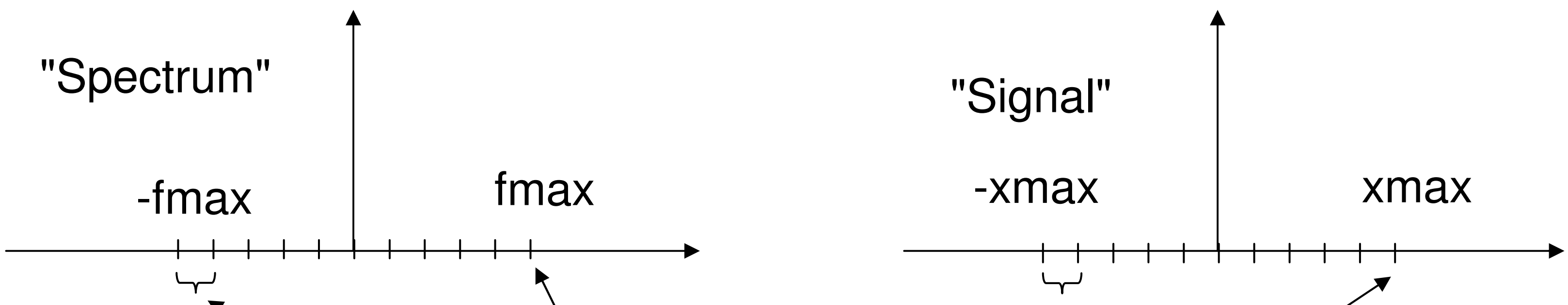
FWHM_x, FWHM_y determine the minimal pixel size

Pixel Size

- Given MxN samples in Fourier space, one can reconstruct MxN pixels using inverse FT
- Pixel size:
  - $\delta x = FOV_x/M = 1/(M\delta u) = 1/U$
  - $\delta y = FOV_y/N = 1/(N\delta v) = 1/V$

Resolution and field of view
1D Signal: Review

Frequency spectrum ⟵ **FT** ⟶ Time series



$f_{max} = f_{s,x}/2 = 1/(2\,\Delta x)$

$x_{max} = f_{s,f}/2 = 1/(2\,\Delta f)$

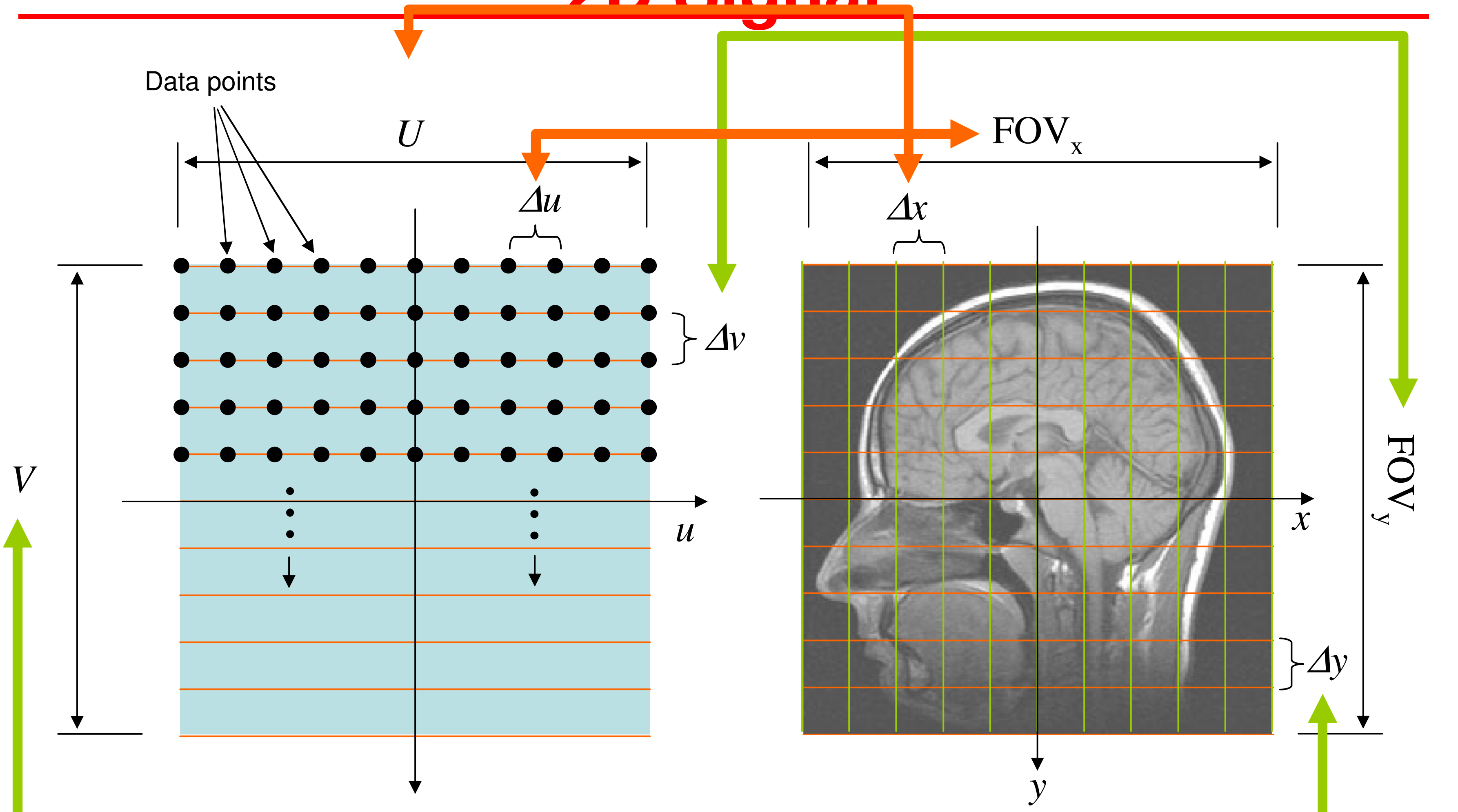
$\Delta f,\ f_{max}$

$\Delta x,\ x_{max}$

$FOVx = 2x_{max} = 1/\Delta f$

Adapted from Graber BMI0 F05 lecture note

Resolution and field of view
2D Signal



$$FOV_x = \frac{1}{\Delta u} = \frac{1}{\bar{\gamma} G_x T}$$

$$FOV_y = \frac{1}{\Delta v} = \frac{1}{\bar{\gamma} \Delta A_y}$$

$$\Delta x = \frac{1}{FOV_u} = \frac{1}{U} = \frac{1}{N_x \bar{\gamma} G_x T} = \frac{1}{\bar{\gamma} G_x T_s}$$

$$\Delta y = \frac{1}{FOV_y} = \frac{1}{V} = \frac{1}{N_y \bar{\gamma} \Delta A_y}$$

Adapted from Graber BMI0 F05 lecture note

Resolution and FOV

- FOV depends on spacing of data points in k-domain (\delta u, \delta v)

$$FOV_x = \frac{1}{\Delta u} = \frac{1}{\bar{\gamma} G_x T}$$

$$FOV_y = \frac{1}{\Delta v} = \frac{1}{\bar{\gamma} \Delta A_y}$$

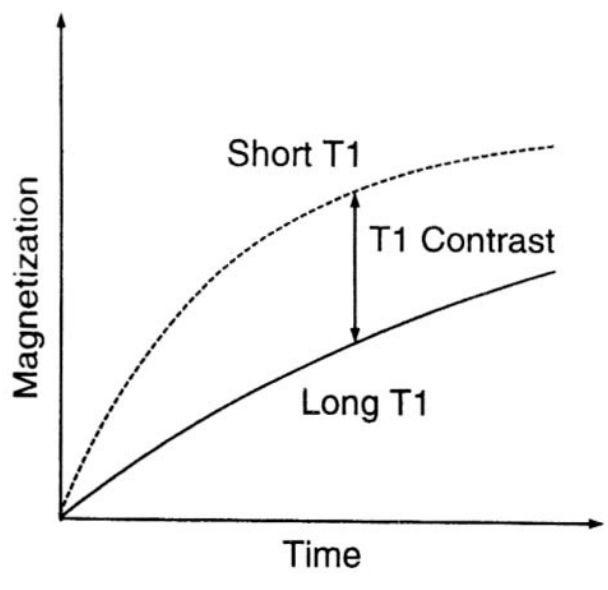
- Resolution (\delta x, \delta y) depends on highest observed spatial frequency component (U, V)

$$\Delta x = \frac{1}{FOV_u} = \frac{1}{U} = \frac{1}{N_x \bar{\gamma} G_x T} = \frac{1}{\bar{\gamma} G_x T_s}$$

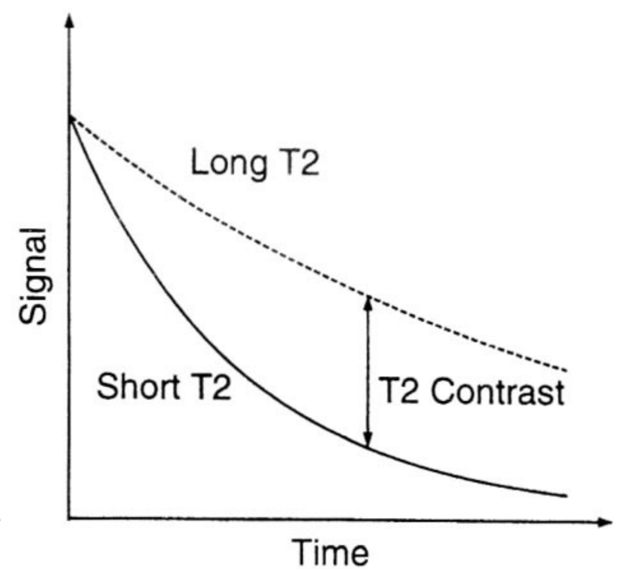
$$\Delta y = \frac{1}{FOV_y} = \frac{1}{V} = \frac{1}{N_y \bar{\gamma} \Delta A_y}$$

Contrast

- Intrinsic :
  Relaxation times T1, T2, proton density, chemical shift, flow
- Extrinsic:
  TR, TE, flip angle
- Contrast in T1:



Contrast in T2:



Noise

- Noise arises from statistical fluctuations of the signal sensed by the receiver coil
- Dominated by Johnson noise – Thermal agitation of electrons or ions in a conductor

$$\sigma^2 = \frac{2k\mathcal{T}R}{T_A}$$

$k =$ Boltzmann's constant

- $\mathcal{T}$ = temperature $\Rightarrow$ colder is better
- R = effective resistance $\Rightarrow$ use small coils
- T_A = total acquisition time $\Rightarrow$ scan longer

R is mainly due to patient body seen by RF coil

SNR

- Recall magnitude of signal is

$$|V| = 2\pi\nu_0 V_s M_0 \sin\alpha B^r$$

- Signal-to-noise Ratio is

$$\begin{aligned} \text{SNR} &= \frac{|V|}{\sqrt{\sigma^2}} \\ &= \frac{\gamma h^2}{\sqrt{4\pi k}} \frac{2\pi\nu_0 P_D \sqrt{\rho}}{r_0^2 \sqrt{L\mathcal{T}^3}} V_s \sin\alpha \sqrt{T_A} \end{aligned}$$

Increase Vs -> thicker slice, larger pixel (but reduced resolution)
Alpha = pi/2
Increase scanning read out time

Advanced MRI Methods

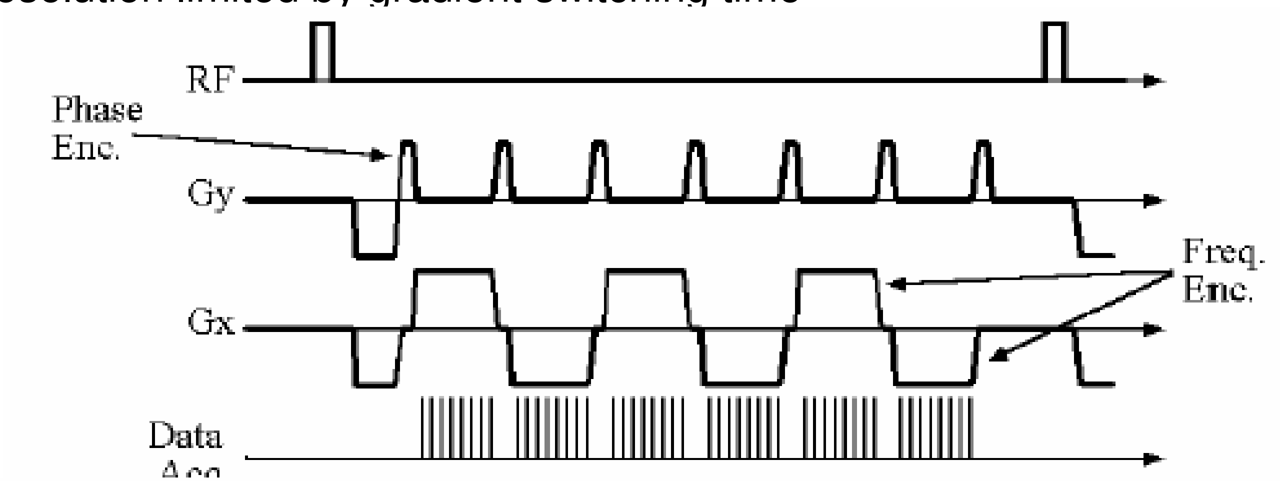
- Multi-slice imaging
- Fast imaging
  - Measuring FID
  - Obtain all phase angles within one RF excitation
- Spiral imaging

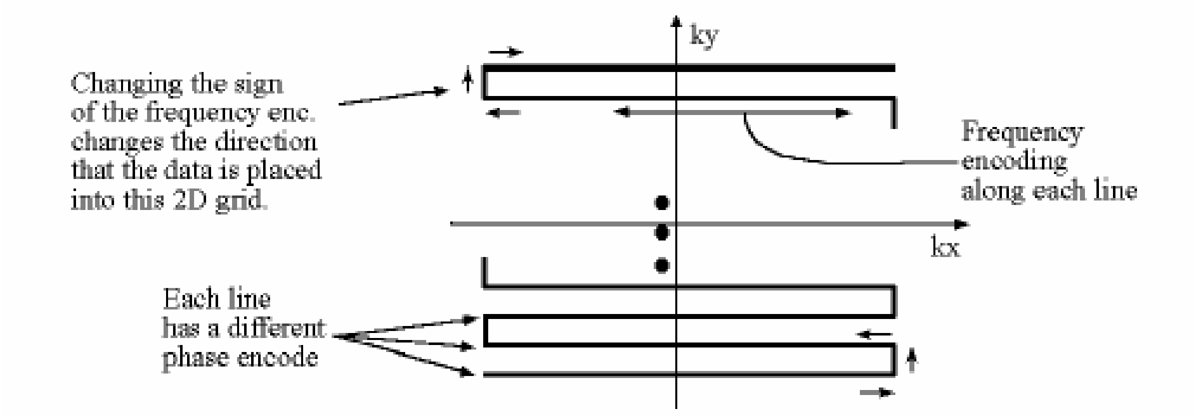
- Functional MRI (fMRI)
  - Used to determine which area of brain is involved in which specific cognitive task
  - T2 and T2* increase locally in areas of brain with neuronal activation, leading to increased signal intensity than normal
- Magnetic resonance angiography (imaging blood flow)
- NMR spectroscopy

- See Webb [Handout]

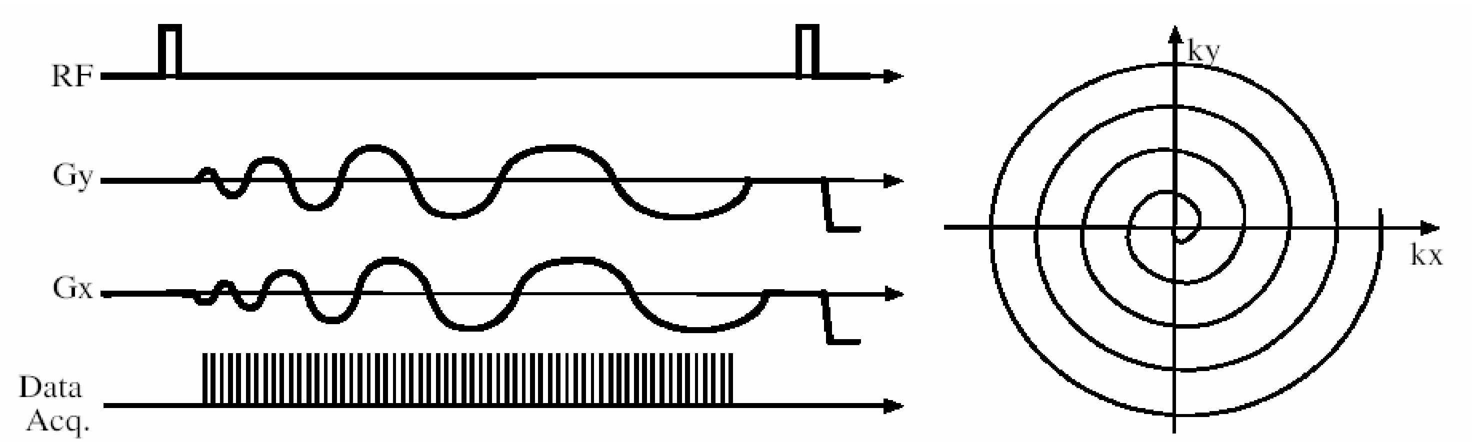
Echo planar imaging

- Avoid going back to origin after each read-out
- “Single shot” imaging, popular in fMRI
- Spatial resolution limited by gradient switching time





Spiral imaging



Clinical Applications of MRI

- Contrast agent (changing T1, T2)
- Brain
  - Brain tumor: increased PD, T1, T2
  - Parkinson's disease, Alzheimer's disease
    - Deposition of iron in the putamen -> reduce T2, T2*
- Liver and the reticuloendothelial system
- Musculoskeletal system
  - Spine, knee, shoulder
- Cardiac system
  - Can differentiate among flowing blood, walls of vessel and cardiac chamber

- See Webb [Handout]

Summary

- MRI data for a slice are Fourier transform of effective spin density distribution
- Reconstruction by inverse FT (rectilinear scan) or filtered backprojection (polar scan)
- Image quality
  - Sampling intervals \delta u, \delta v determine the field of view in the signal domain
    - Small \delta u, \delta v -> larger field of view
  - Coverage area U, V determine blurring
    - Larger coverage area -> narrower blurring function (better resolution)
  - Noise level
    - Dominated by Johnson noise
  - SNR
    - Better with stronger static magnetic field, and longer read-out time (but can reduce spatial resolution)

Reference

- Prince and Links, Medical Imaging Signals and Systems, Chap. 13
- A. Webb, Introduction to Biomedical Imaging, Chap. 4
- **The Basics of MRI**, A web book by Joseph P. Horn (containing useful animation):
- http://www.cis.rit.edu/htbooks/mri/inside.htm

Homework

- Reading:
  - Prince and Links, Medical Imaging Signals and Systems, Chap. 13
  - Webb, Introduction to biomedical imaging, Sec. 4.8-4.12
  - Note down all the corrections for Ch. 12,13 on your copy of the textbook based on the provided errata (see Course website or book website for update).
- Problems
  - P13.19
  - P13.20
  - P13.26
  - P13.28