Parametric instabilities

Name comes from systems with periodically varying parameters
E.g. spring with vertically oscillating suspension point, spring with periodic variation of stiffness. For pendulum X would be changed to sin(X).

$$\frac{\mathrm{d}^2 X}{\mathrm{d}t^2} + \Omega_0^2 \left[1 - 2\varepsilon \cos\left(\omega_0 t\right)\right] X = 0$$

Mathieu equation

Amplitude of X for small ε grows, when $\Omega_0 = n\omega_0/2$, simplest for n=1 $\Rightarrow$ $\omega_0 \rightarrow \Omega_0 + \Omega_0$, the range of unstable frequencies increases with growth of ε

Plasma dynamics and Maxwell's equations include nonlinear terms

$$\vec{j}_s = q_s n_s \vec{\mathrm{v}}_s$$

n_s can have frequency ω_1 and v_s frequency ω_2

$$\vec{j}_s = q_s \frac{1}{2}\left(n_1 \mathrm{e}^{-i\omega_1 t} + n_1^* \mathrm{e}^{i\omega_1 t}\right)\frac{1}{2}\left(\vec{\mathrm{v}}_2 \mathrm{e}^{-i\omega_2 t} + \vec{\mathrm{v}}_2^* \mathrm{e}^{i\omega_2 t}\right) =$$

$$= \frac{q_s}{4}\left(n_1 \vec{\mathrm{v}}_2 \mathrm{e}^{-i(\omega_1+\omega_2)t} + n_1^* \vec{\mathrm{v}}_2^* \mathrm{e}^{i(\omega_1+\omega_2)t}\right) + \frac{q_s}{4}\left(n_1 \vec{\mathrm{v}}_2^* \mathrm{e}^{-i(\omega_1-\omega_2)t} + n_1^* \vec{\mathrm{v}}_2 \mathrm{e}^{i(\omega_1-\omega_2)t}\right)$$

Suggested reading - Chen 8.5, Nicholson 7.17

Current thus contains **sum and difference frequencies**

If strong wave E_0 with frequency ω_0 and wave vector $\vec{k}_0$ is in plasma

n_1 weak random perturbation with $\omega_1, \vec{k}_1$ $\Rightarrow$ current $\omega_0 - \omega_1, \vec{k}_0 - \vec{k}_1$

if it is in resonance with wave $\omega_2, \vec{k}_2$, it pumps it $\omega_2 = \omega_0 - \omega_1, \quad \vec{k}_2 = \vec{k}_0 - \vec{k}_1$

combination of waves 0,2 pumps perturbation 1 $\omega_1 = \omega_0 - \omega_2, \quad \vec{k}_1 = \vec{k}_0 - \vec{k}_2$

parametric instability is usually stimulated decay – most of energy goes to wave with higher frequency $\Rightarrow \hbar\omega_0 = \hbar\omega_1 + \hbar\omega_2$

Also, **ponderomotive force** participates in parametric amplification
Force is proportional to square of electric field – it contains non-linear terms $\sim E^*_{\omega_1} E_{\omega_2}$

Let's assume laser wave $\vec{E}_d = \hat{x} E_0 \exp\left(-i\omega_0 t\right)$ ($\vec{k}_0 \parallel \hat{z}$ small)

Let's small slowly varying density perturbation n_p has $\vec{k} \parallel \vec{E}_0$

$n = n_0 + n_p = n_0 + \Delta n \cos\left(kx\right)$ due to quasineutrality $n=n_e=Zn_i$

Due to electron oscillations in the direction of density gradient there appears fast varying perturbation of electron density $\tilde{n}_l \sim \Delta n \cdot E_0$ - longitudinal wave

Electron density $n = n_0 + n_p + \tilde{n}_l$, velocity $u = u_0 + \tilde{u}_l$, electric field $E = E_0 + \tilde{E}_l$

Continuity equation for electrons (u_0 – electron oscillation velocity)

$$\frac{\partial n}{\partial t} + \mathrm{div}\left(n\vec{u}\right) = 0$$ has the form $$\frac{\partial \tilde{n}_l}{\partial t} + n_0 \frac{\partial \tilde{u}_l}{\partial x} + u_0 \frac{\partial n_p}{\partial x} = 0$$

Equation of motion

$$\frac{\partial \vec{u}}{\partial t} + \left(\vec{u}\nabla\right)\vec{u} = -\frac{\nabla p_e}{m_e n_e} - \frac{e\vec{E}}{m_e}$$ transfers to the form $$\frac{\partial \tilde{u}_l}{\partial t} = -\frac{\nabla \tilde{p}_l}{m_e n_0} - \frac{e\tilde{E}_l}{m_e}$$

and Poisson equation $\frac{\partial \tilde{E}_l}{\partial x} = -\frac{e}{\varepsilon_0}\tilde{n}_l$

$\Rightarrow \left(\omega_0^2 - \omega_{pe}^2 - 3k^2 \mathrm{v}_{Te}^2\right)\tilde{E}_l = \omega_{pe}^2 n_p E_0 / n_0$ term $\sim n_\mathrm{p} E_0$ source of l wave

And from this $\tilde{E}_l = \frac{\omega_{pe}^2}{\omega_0^2 - \omega_{pe}^2 - 3k^2 \mathrm{v}_{Te}^2} \frac{\Delta n}{n_0} E_0 \cos kx$

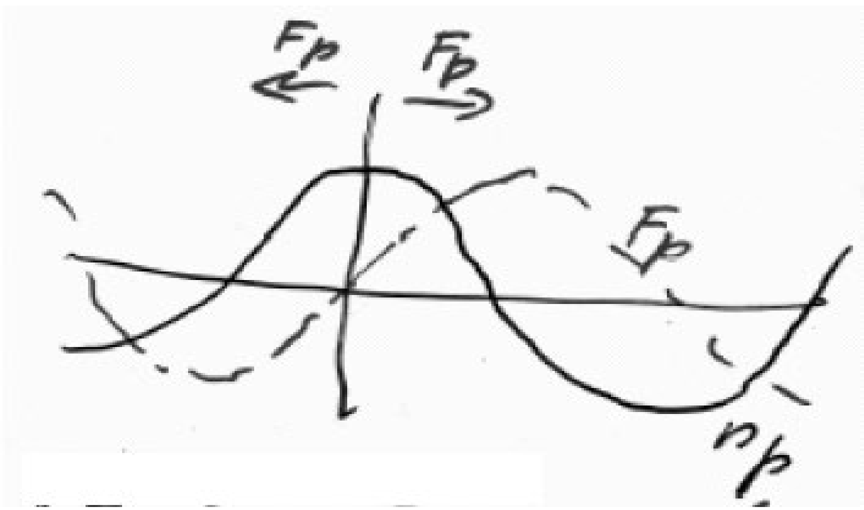
Let's denote $\omega_{ek}^2 = \omega_{pe}^2 + 3k^2 \mathrm{v}_{Te}^2$

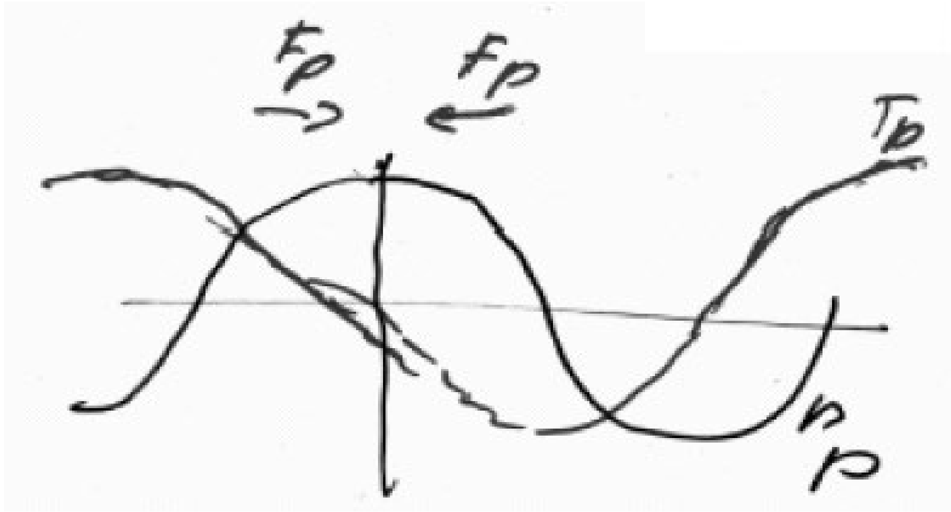
and ponderomotive force (with use $\left|\tilde{E}_l\right| \ll \left|E_0\right|$) is

$$f_p \sim -\nabla \left|E\right|^2 = -\nabla \left|E_0 + \tilde{E}_l\right|^2 = -\nabla\left(E_0^* \tilde{E}_l + E_0 \tilde{E}_l^*\right) = \frac{\omega_{pe}^2}{\omega_0^2 - \omega_{ek}^2} \frac{\Delta n}{n_0} E_0^2 k \sin kx$$

Low frequency ponderomotive force acts on electrons and ions and it modifies initial density perturbation n_p

for $\omega_0 > \omega_{ek}$ for $\omega_0 < \omega_{ek}$





pond. force (F_p) decreases density perturbation - **stable** for exp. growth

F_p increases density perturbation **instability** (aperiodic) – **oscillating two-stream instability**

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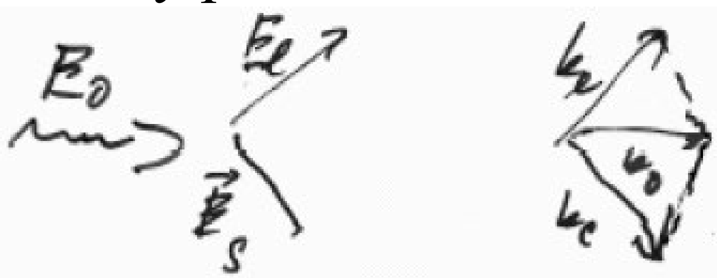
density perturbation $\Rightarrow$ ion acoustic wave, resonance condition $\omega_s = k_s c_s$



$$\omega_0 = \omega_l + \omega_s$$

$$\vec{k}_0 = \vec{k}_l + \vec{k}_s$$

(parametric decay instability)

Parametric instabilities in plasmas

a) $p \rightarrow p' + s$ plasmon decay

b) $l \rightarrow p + s$ photon decay – parametric decay instability (PRN) and oscillating two-stream (aperiodic) instability (ODN) - absorption of laser radiation

c) $l \rightarrow l' + s$ SBS (stimulated Brillouin scattering) - reflection

d) $l \rightarrow l' + p$ SRS (stimulated Raman scattering) – both reflection and absorption

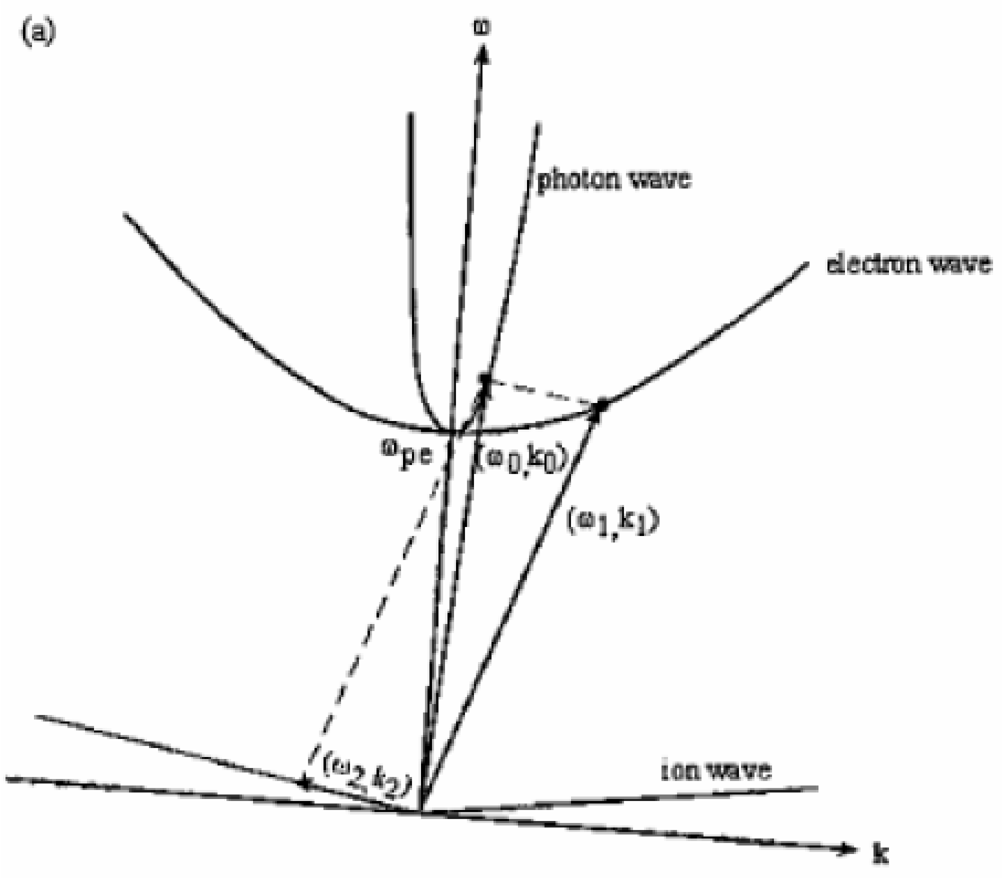
e) $l \rightarrow p + p'$ TPD (two-plasmon decay DPRN) –absorption

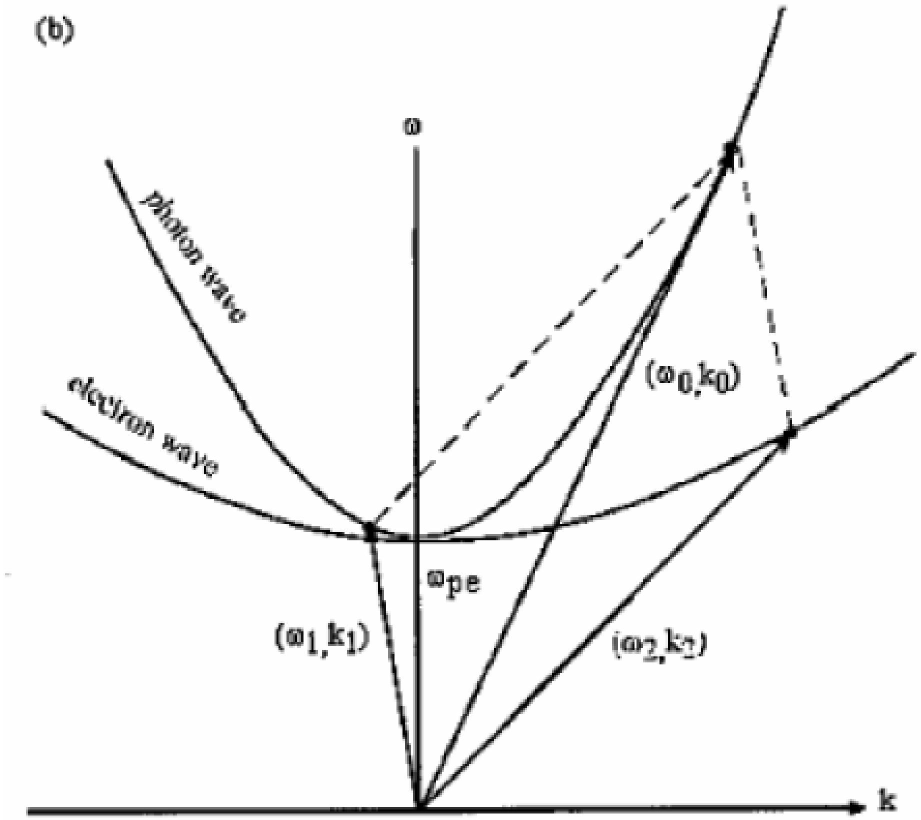
The pump wave has to overcome damping of the daughter waves. A threshold exists for each instability, under which plasma is stable.

The threshold for instabilities stimulated by laser radiation depends on parameter $I\lambda^2$, so the threshold rises when wavelength is decreased

For Nd laser (λ = 1,06 μm) the threshold for interaction of nanosecond pulses with solid targets is typically ~10^{13}- 10^{14} W/cm^2. 3rd harmonic frequency of Nd-laser is usually used in fusion experiments to avoid problems with parametric instabilities.

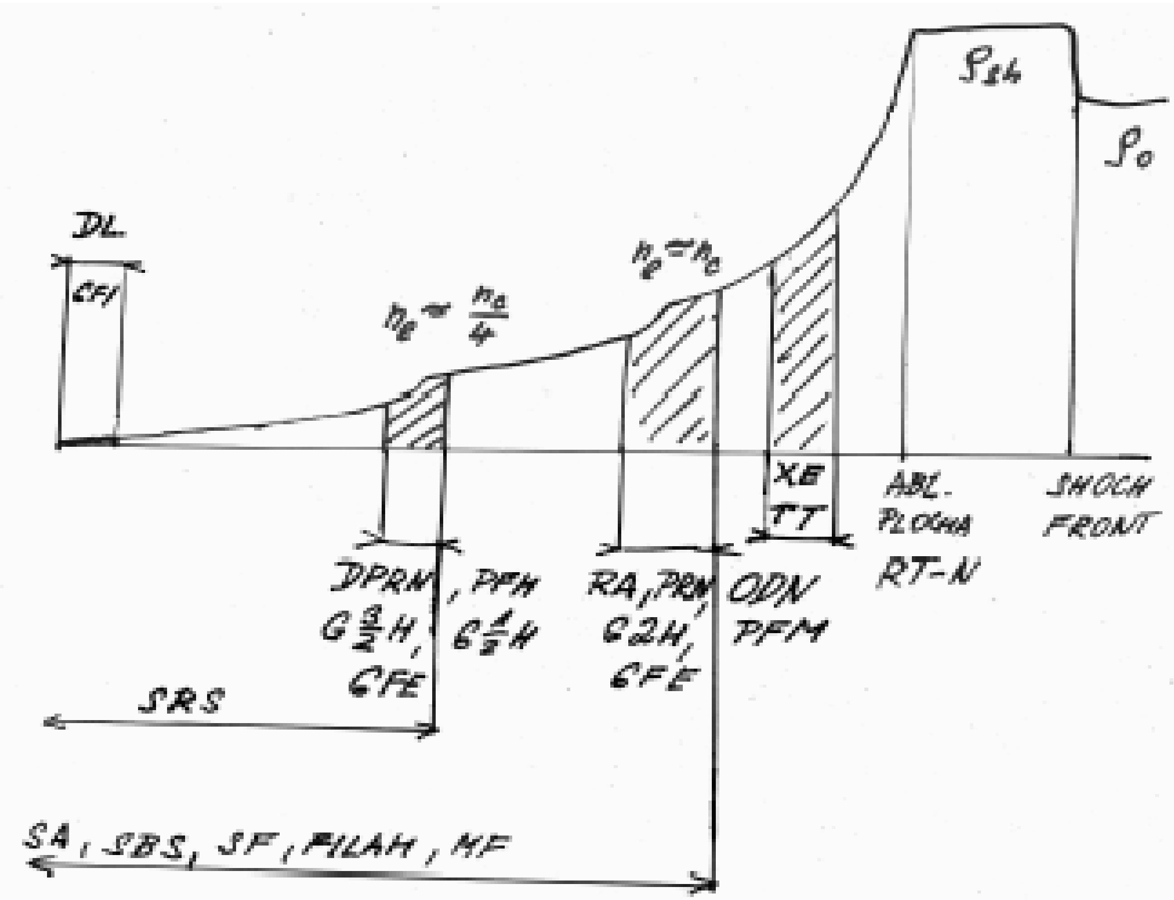
Synchronization conditions for (a) parametric decay instability, (b) stimulated Raman scattering





Pump wave must meet synchronization conditions and its intensity must be above threshold (often given by the damping of daughter waves).

Physics of corona and laser interaction with plasma (target density ρ_0)



DL – sheath (double layer)
GFI – ion acceleration
DPRN – two-plasmon decay
SA – collisional absorption (inverse bremsstrahlung)
SF – self-focusing
FILAM – laser beam filamentation
GFE – electron acceleration
PFM – density profile modification by ponderomotive force
GxH – generation of x^{th} harmonics
RA – resonance absorption
XE – X-ray generation
PRN – parametric decay instability
TT – heat transport
RT-N – Rayleigh-Taylor instability
Shock front, MF – generation of spontaneous magnetic fields
SBS –stimulated Brillouin scattering, SRS - stimulated Raman scattering

Conditions for ICF (inertial confinement fusion) are (1) high absorption and hydrodynamic efficiencies, (2) minimum fast electrons, (3) homogeneous ablation on ablation surface