INSTITUTE OF PLASMA PHYSICS
OF THE CZECH ACADEMY OF SCIENCES

INTRODUCTION TO NEUTRAL BEAM INJECTION (NBI) IN TOKAMAKS: NBI MODELLING

FABIEN JAULMES

Institute of Plasma Physics of the CAS, Prague, Czech Republic

MINISTRY OF EDUCATION,
YOUTH AND SPORTS

Overview of this lecture

- What is the NBI [Neutral Beam Injector]
- Modelling particle orbits in tokamaks
- Coulomb Collisions and slowing down distribution
- Application: modelling of fast neutrons generation in COMPASS Upgrade

NBI IN SUPPORT OF TOKAMAK PLASMA

Tokamak plasma **heating**

Tokamak plasma **current drive**

Neutral beam injection:

- Fast **neutrals** injected (**10-1000 keV**) into tokamak (no interaction with tokamak magnetic field)
- **Ionized** by collisions with plasma particles
- **Fast ions** slow down by **Coulomb collision** and transfer their energy to plasma particles

Positive ions based beam → accelerated positive molecular ions **(D_2^+, D_3^+, D_2O^+)** → fractional energies **(E/2, E/3, E/10)**

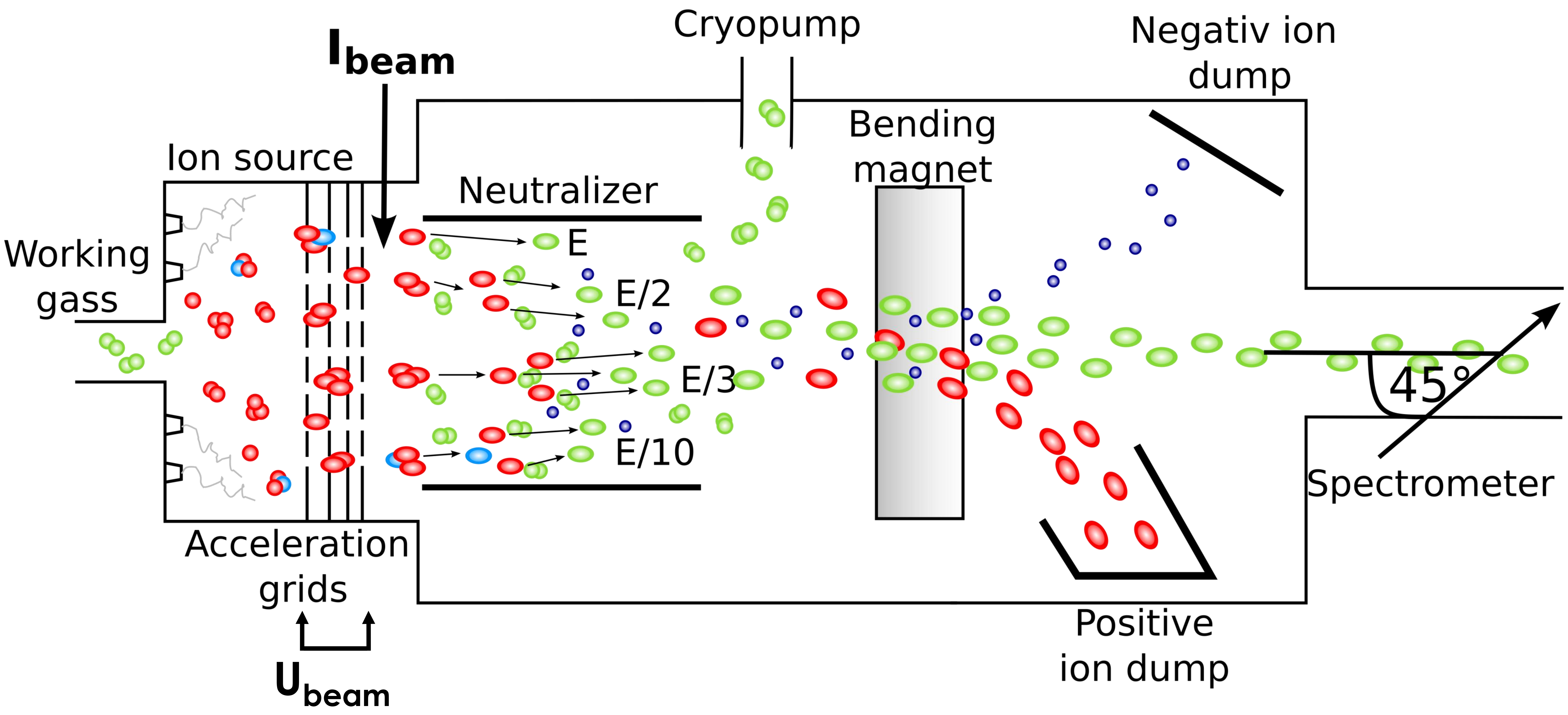
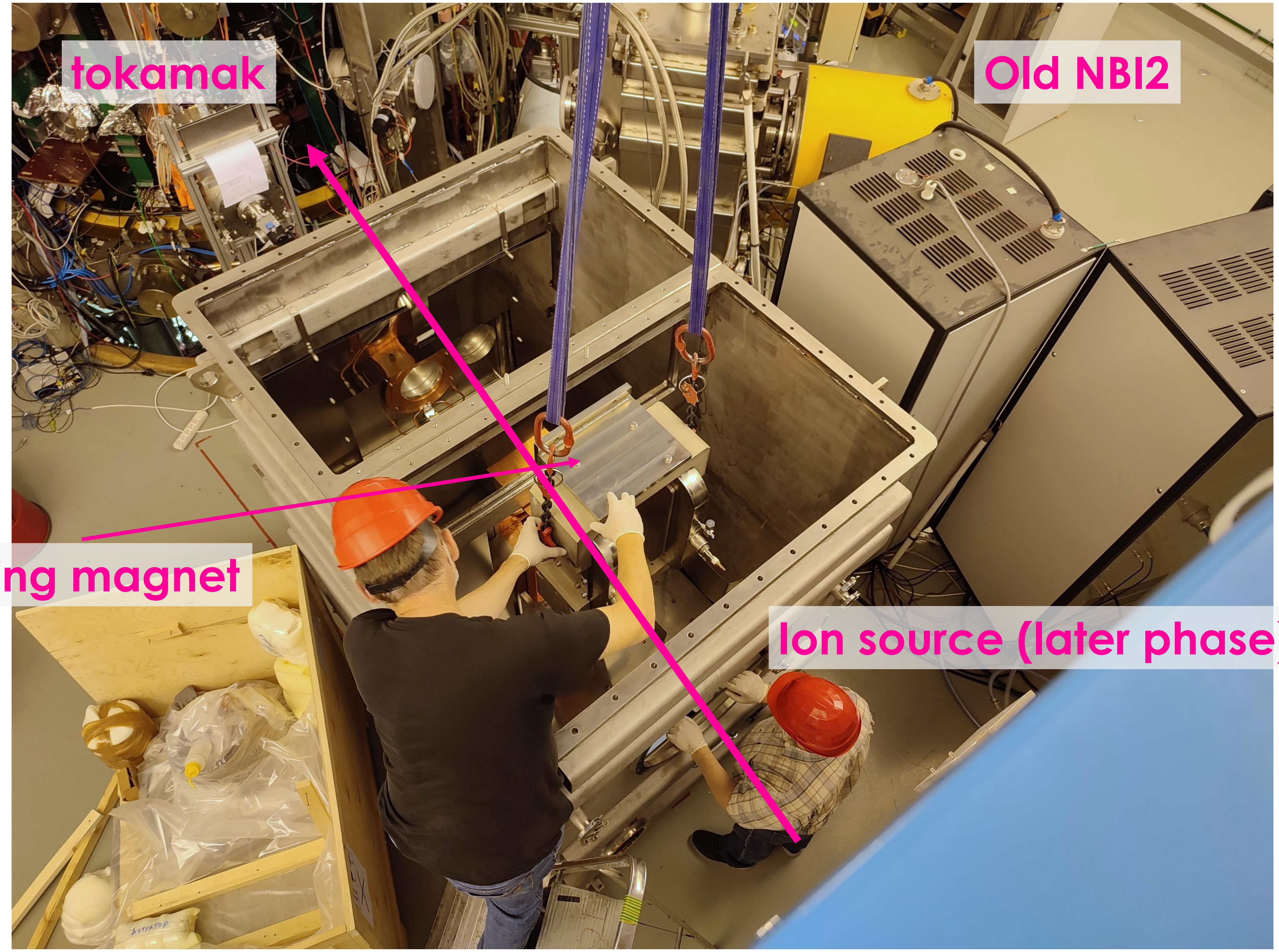
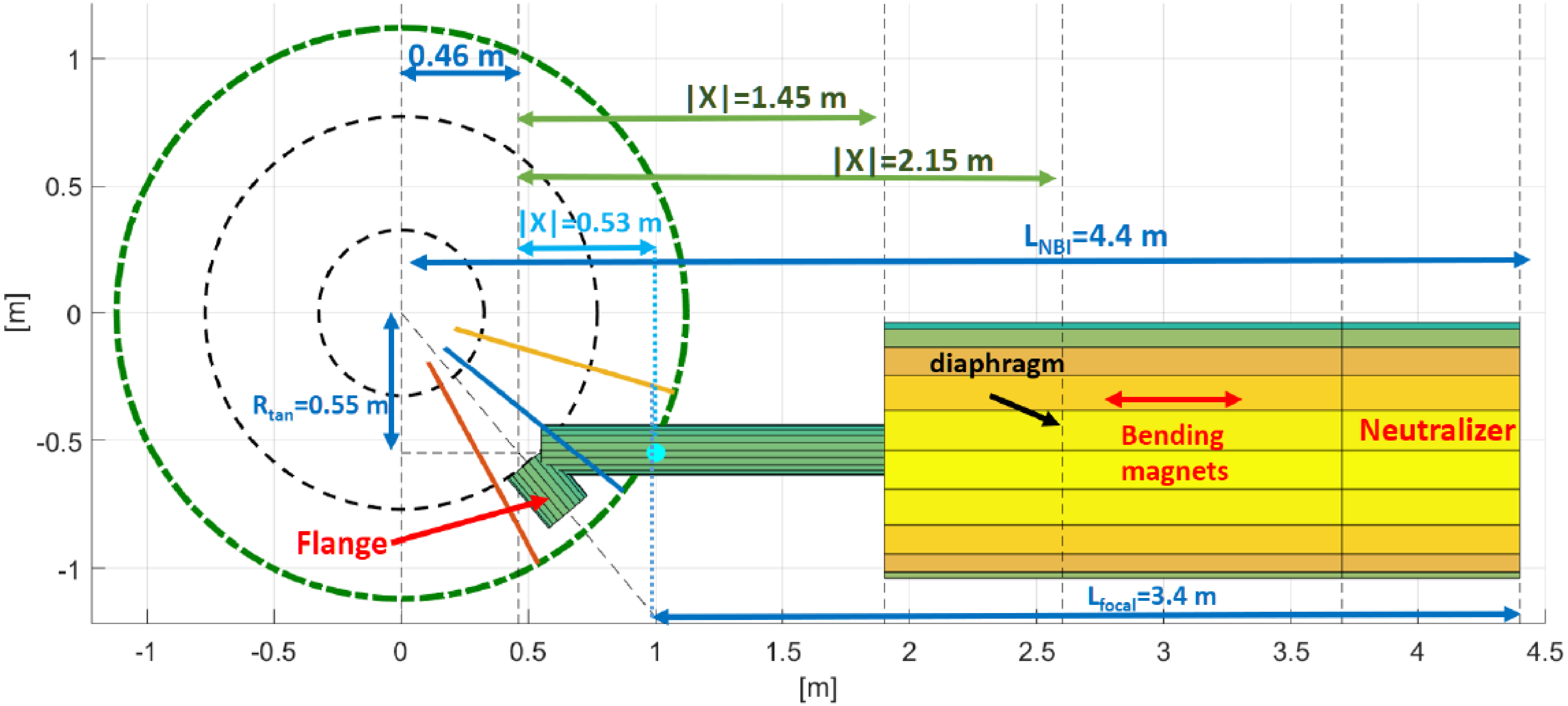


PHOTO OF COMPASS INJECTORS



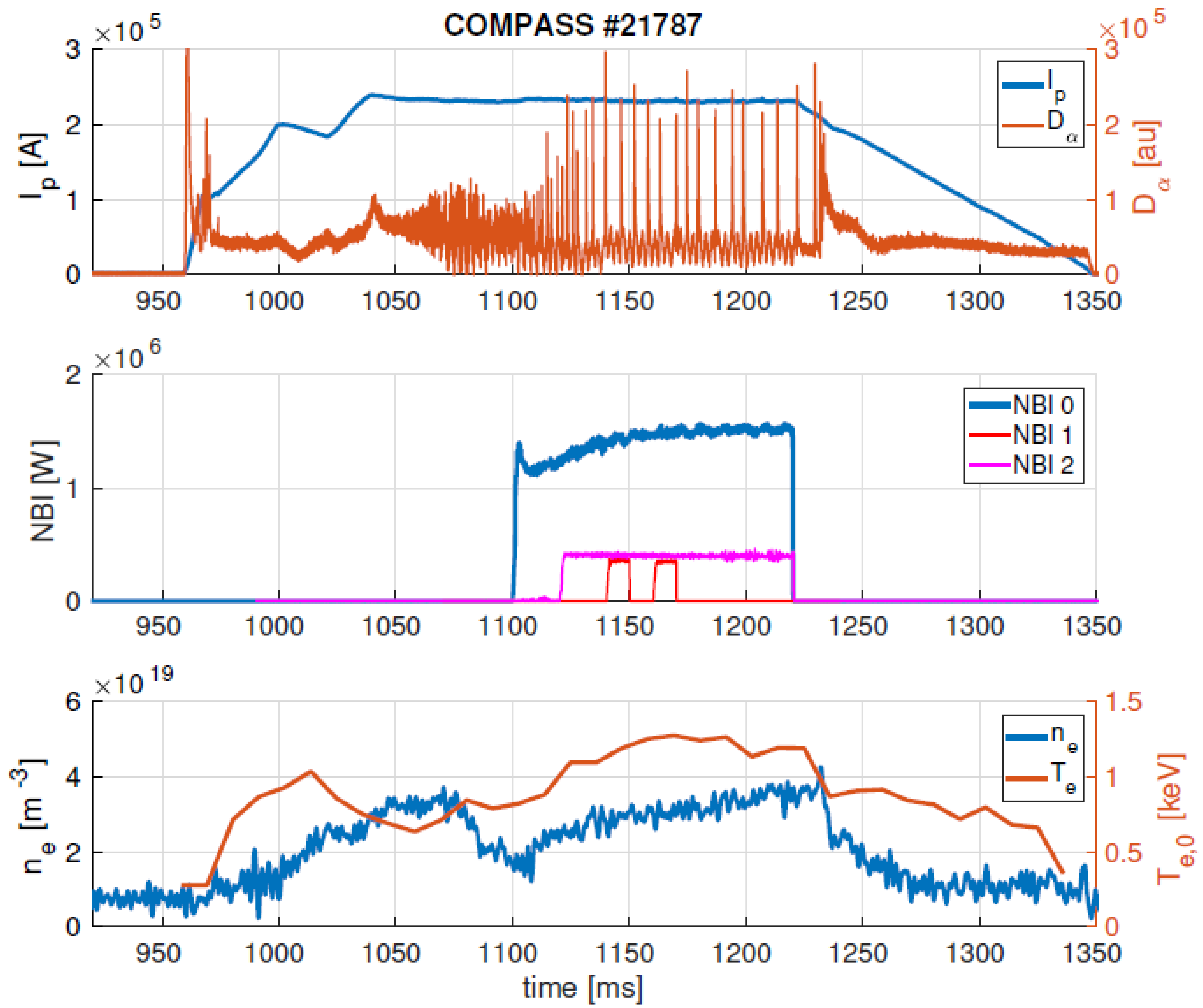
Overview of the dimensions of the NBI 0 in COMPASS\ [top view]



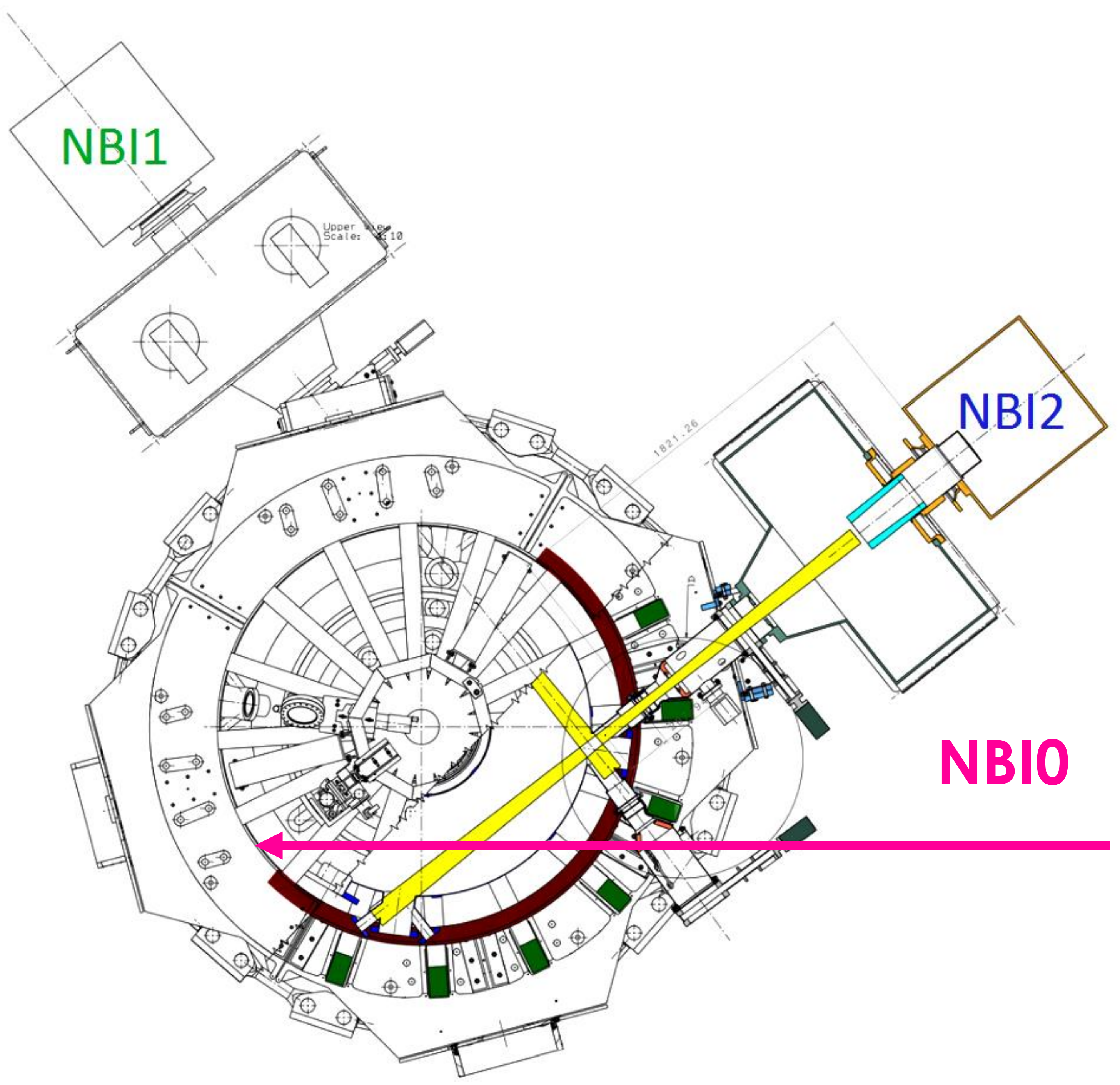
F. Jaulmes et al 2022 PPCF 64 125001

COMPASS EXTERNALLY HEATED PLASMA

New 1 MW NBI-0 in COMPASS allowed easier access to ELMy H-mode [higher confinement mode]



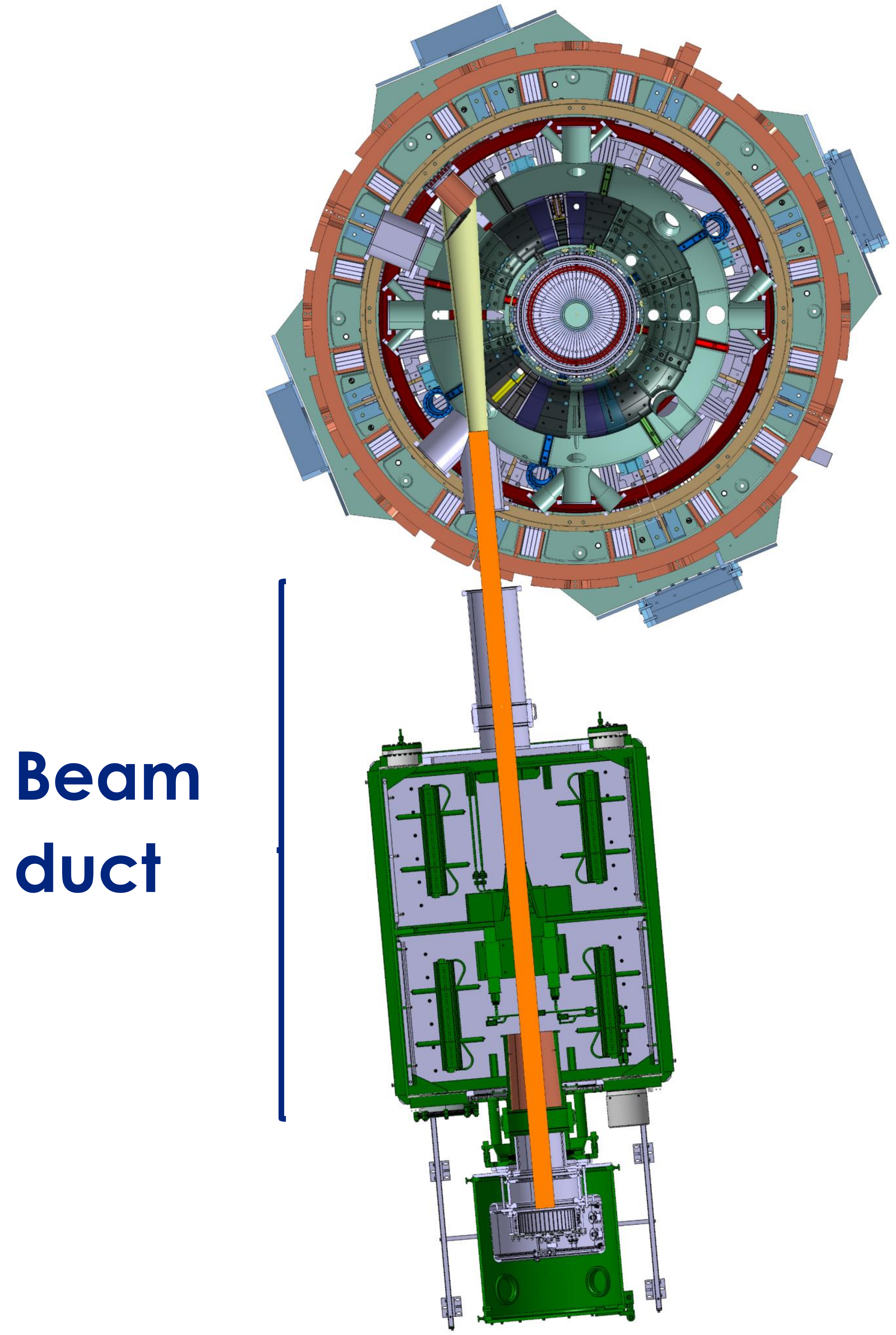
	NBI0	Old NBIs (NBI1 + NBI2)
NBI0 current I_{beam}	≤ 22 A (-> power ~1MW)	≤ 12 A (->power 400 kW)
NBI0 acc. Voltage U_{beam}	≤ 80 keV (70 keV in experiments)	≤ 40 keV
NBI0 pulse duration	≤ 1s	≤ 0.3 s
Tangency radius	0.55m	0.55m



	NBI0	Old NBIs (NBI1 + NBI2)
NBI0 current I_{beam}	≤ 22 A (-> power ~1MW)	≤ 12 A (->power 400 kW)
NBI0 acc. Voltage U_{beam}	≤ 80 keV (70 keV in experiments)	≤ 40 keV
NBI0 pulse duration	≤ 1s	≤ 0.3 s
Tangency radius	0.55m	0.55m

Table 2. Neutralization efficiencies $\mathcal{N}_{\mathrm{eff}}$, including dissociation of the molecular ions, of the ions originating from the source and resulting power fractions $\mathcal{P}_{\mathrm{frac}}$ after the neutralizer for the nominal energies of 58 keV (#21760), 66 keV (#21787) and 80 keV.

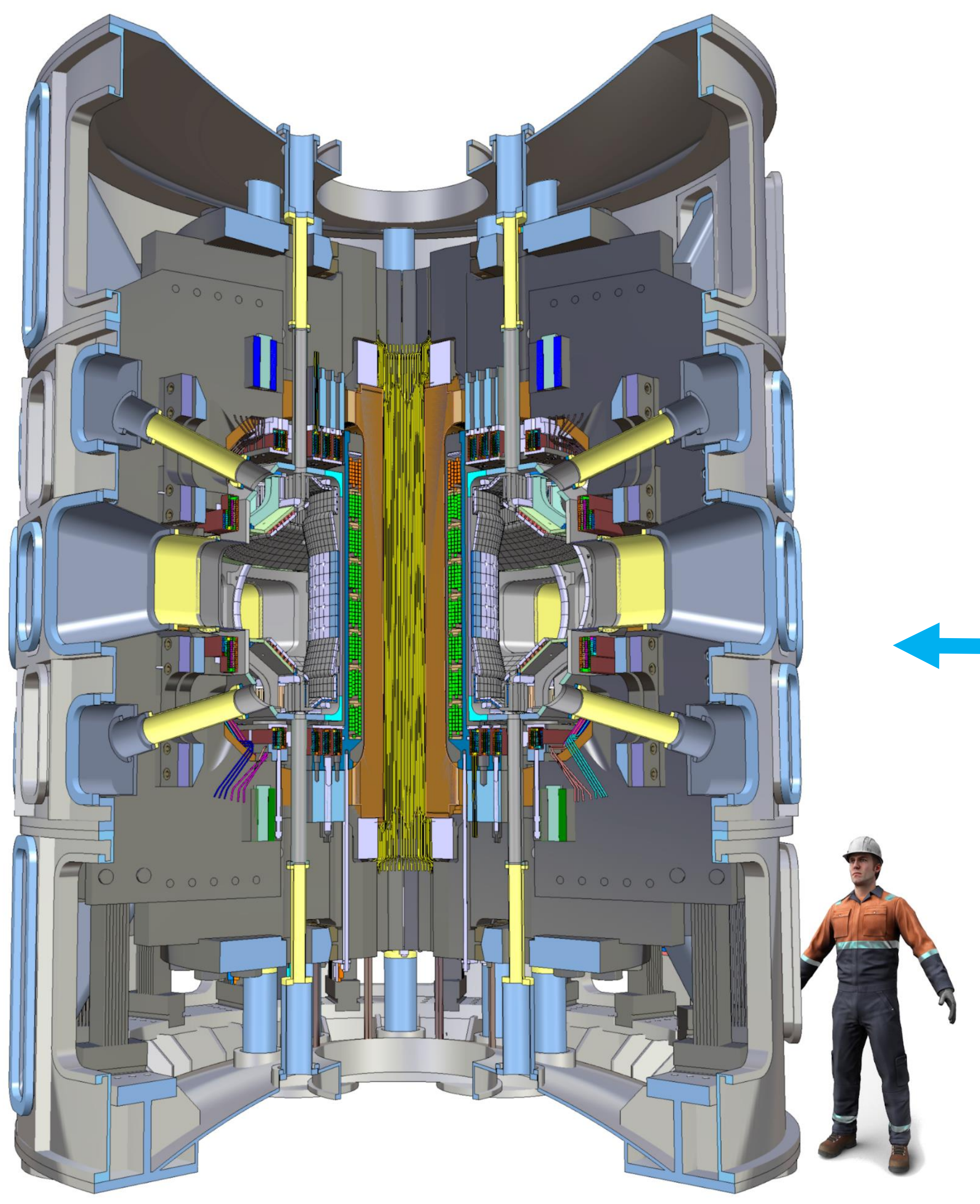
	$\mathcal{E}_0 = 58\,\mathrm{keV}$		$\mathcal{E}_0 = 66\,\mathrm{keV}$		$\mathcal{E}_0 = 80\,\mathrm{keV}$	
	$\mathcal{N}_{\mathrm{eff}}$	$\mathcal{P}_{\mathrm{frac}}$	$\mathcal{N}_{\mathrm{eff}}$	$\mathcal{P}_{\mathrm{frac}}$	$\mathcal{N}_{\mathrm{eff}}$	$\mathcal{P}_{\mathrm{frac}}$
D^+	0.72	0.38	0.68	0.37	0.61	0.35
D_2^+	1.74	0.51	1.71	0.51	1.64	0.52
D_3^+	2.71	0.09	2.69	0.1	2.6	0.1
D_2O^+	1.8	0.02	1.8	0.02	1.8	0.02
Overall	0.76	1.0	0.73	1.0	0.7	1.0



Overview of this lecture

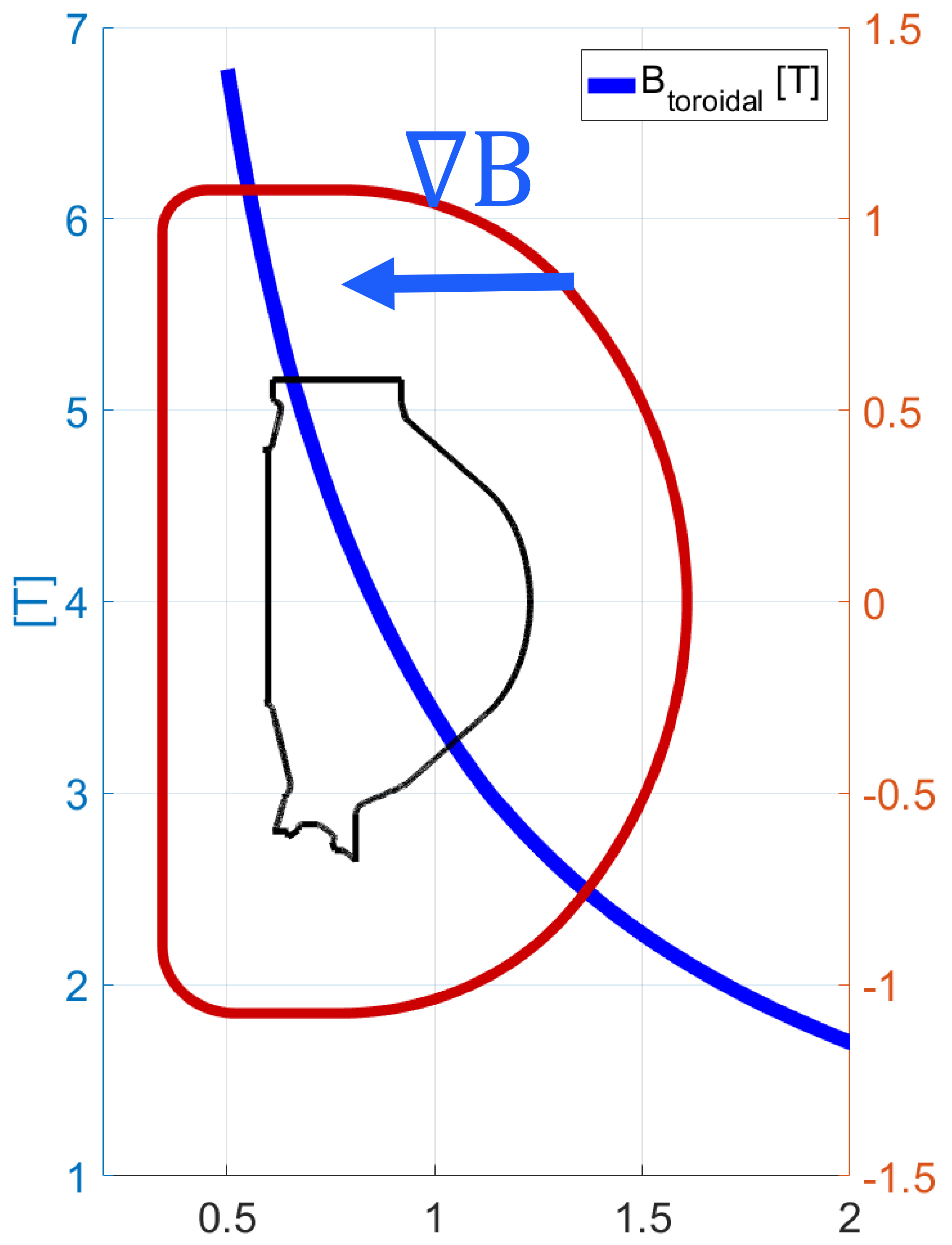
- What is the NBI [Neutral Beam Injector]
- **Modelling particle orbits in tokamaks**
- **Coulomb Collisions and slowing down distribution**
- **Application: modelling of fast neutrons generation in COMPASS Upgrade**

Toroidal device: COMPASS Upgrade



P. Vondracek et al. Fusion Engineering and Design 169 (2018) 112490

COMPASS Upgrade, B_t=5T
Prague 8, Czech Republic



- A toroidal field is created by juxtaposing coils in a circle so that their fields add up to build a circular "toroidal" field

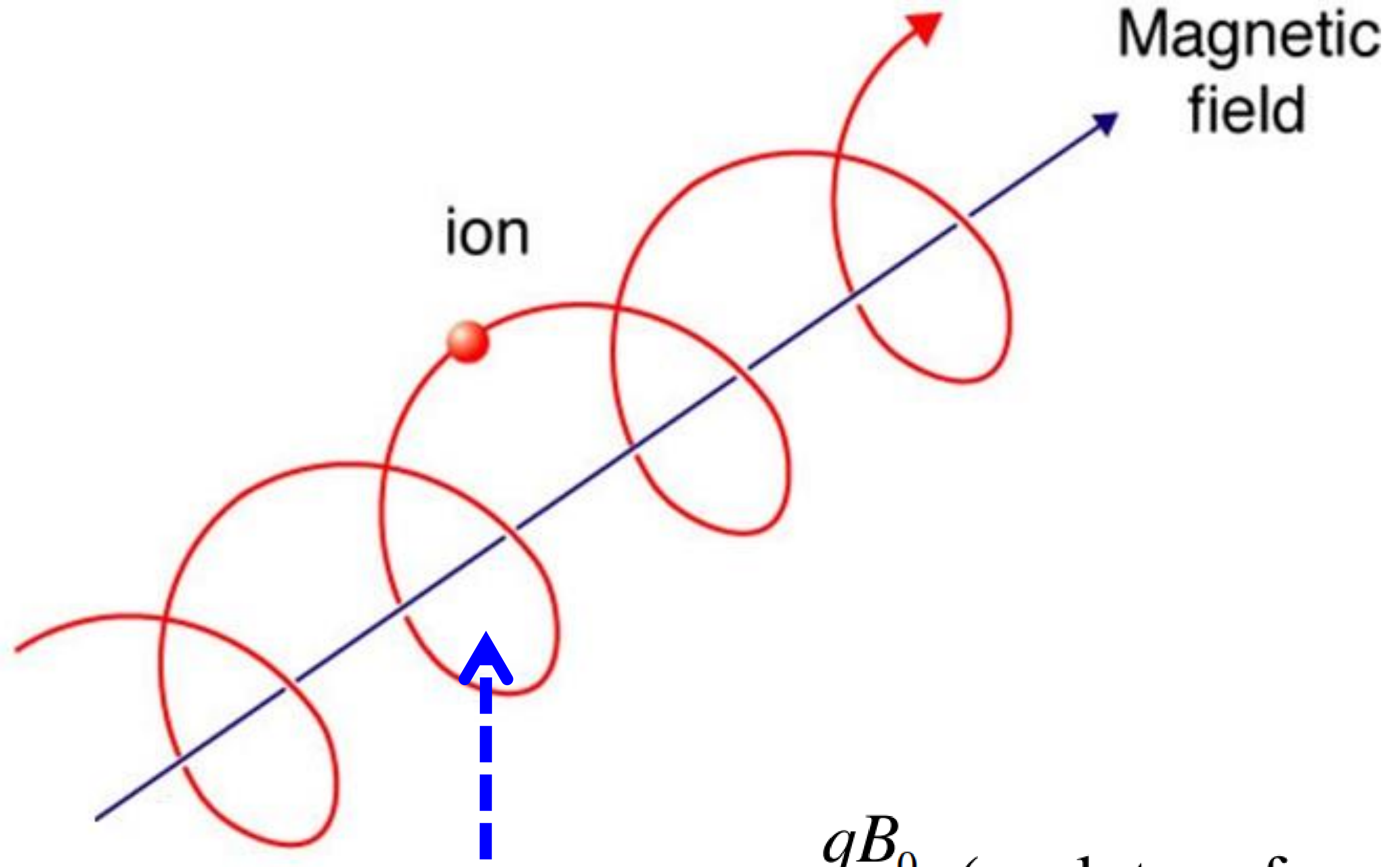
Magnetic confinement

- **Charged particles “follow” magnetic field lines**
- **Their motion is ruled by the Lorentz force**

$$\frac{d^2\mathbf{x}}{dt^2} = \frac{Z_i e}{m}[\mathbf{E} + \mathbf{v} \times \mathbf{B}]$$

Deuterium (D)
m=3.3x10^{-27} kg

Elementary charge: e=1.6x10^{-19} C



$$\omega_c = \frac{qB_0}{m} \text{ (cyclotron frequency)}, f_c = \omega_c / 2\pi$$

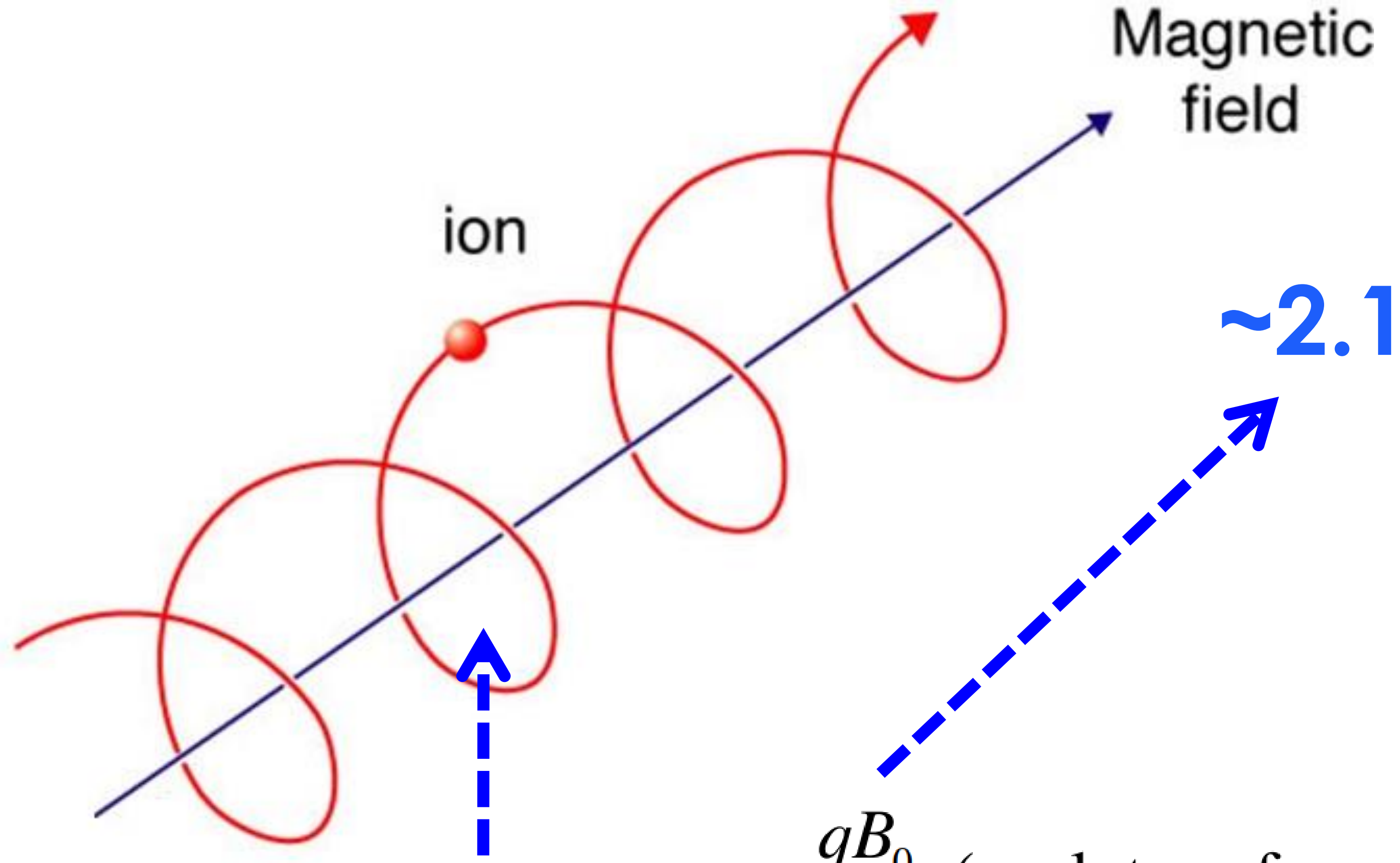
$$r_L = \left|\frac{v_\perp}{\omega_c}\right| = \left|\frac{mv_\perp}{qB_0}\right| \text{ (Larmor radius, gyroradius)}$$

Gyrating motion is called « Larmor orbit »

Magnetic confinement

- Charged particles “follow” magnetic field lines
- Their motion is ruled by the Lorentz force

$$\frac{d^2\mathbf{x}}{dt^2} = \frac{Z_i e}{m}\left[\mathbf{E} + \mathbf{v} \times \mathbf{B}\right]$$



~2.1*10^8 rad/s

$$\omega_c = \frac{qB_0}{m} \text{ (cyclotron frequency)}, \; f_c = \omega_c / 2\pi$$

$$r_L = \left|\frac{v_\perp}{\omega_c}\right| = \left|\frac{mv_\perp}{qB_0}\right| \text{ (Larmor radius, gyroradius)}$$

Deuterium (D)
m=3.3x10^{-27} kg

Elementary charge: e=1.6x10^{-19} C

Gyrating motion is called « Larmor orbit » → **6-17 mm**

COMPASS-U 80keV, 4.3T

Modelling of fast ion trajectories in electromagnetic fields

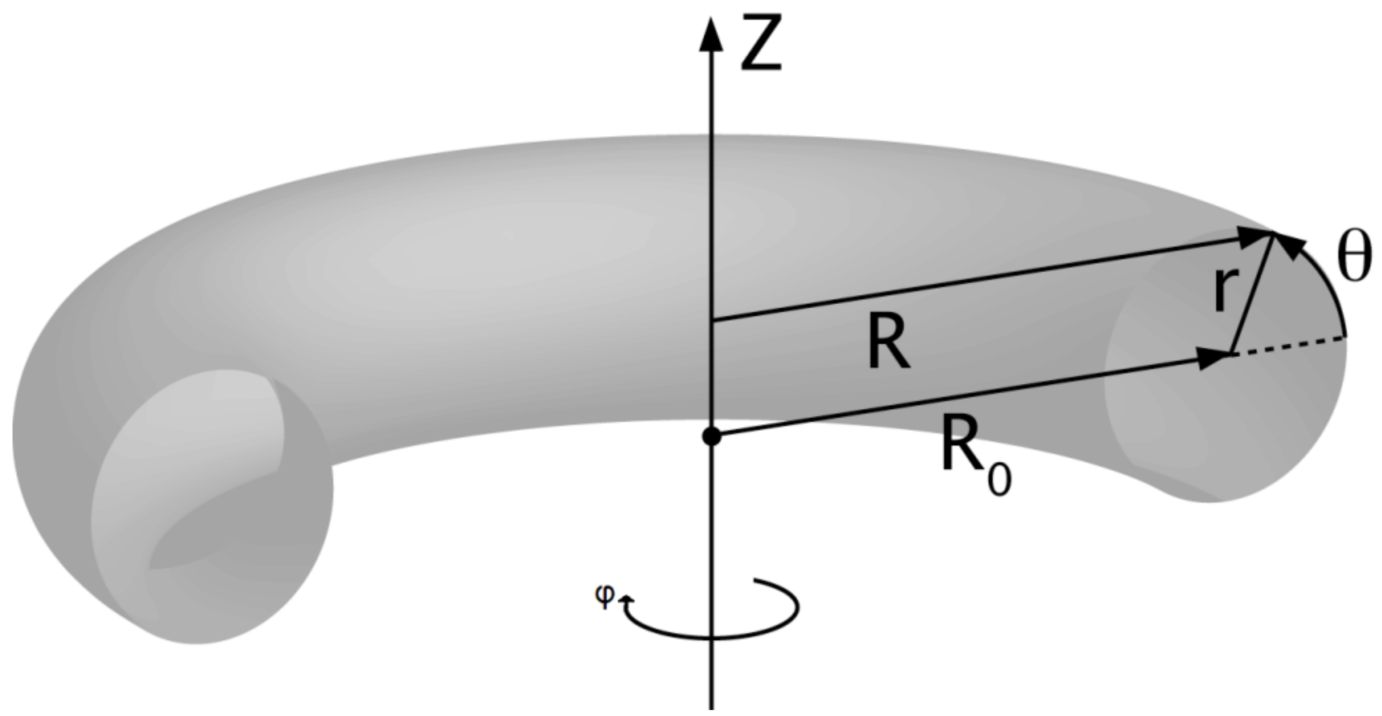
Guiding center approximation:

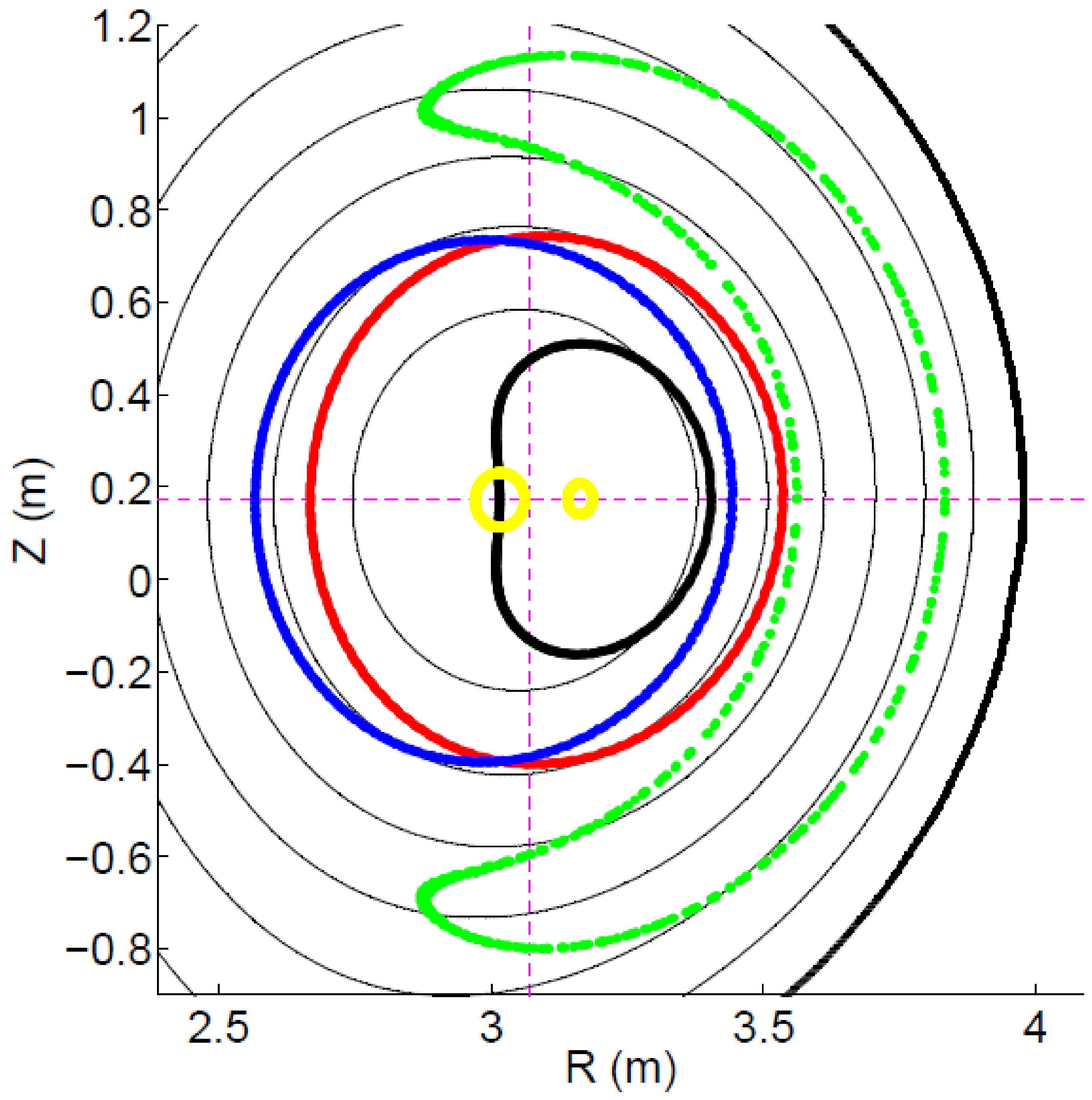
$$\frac{\mathrm{d}\mathbf{X_{gc}}}{\mathrm{d}t} = \mathbf{v}_\parallel + \mathbf{v_E} + \frac{2\mathcal{E}_{kin} - \mu B}{(Z_i e)B^2}(\mathrm{b} \times \nabla B) + \frac{1}{\omega_{ci}}\left(\mathrm{b} \times \frac{\mathrm{d}\mathbf{v_E}}{\mathrm{d}t}\right)$$

Full-orbit description

$$\frac{\mathrm{d}^2\mathbf{x}}{\mathrm{d}t^2} = \frac{Z_i e}{m}[\mathbf{E} + \mathbf{v} \times \mathbf{B}]$$

Tokamak cylindrical coordinates (R,Z,φ)



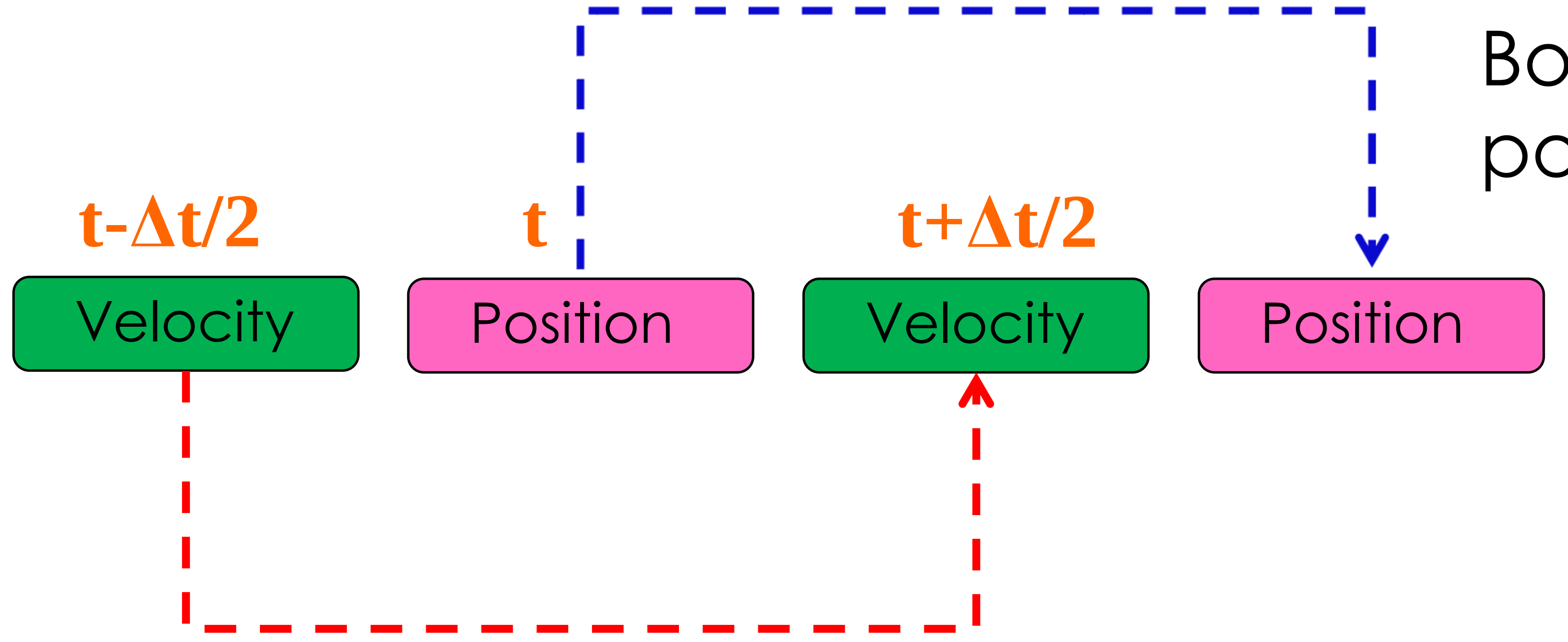


Guiding center trajectories of fast alphas in JET

Modelling of fast ion trajectories in electromagnetic fields

$$\frac{d^2\mathbf{x}}{dt^2} = \frac{Z_i e}{m}[\mathbf{E} + \mathbf{v} \times \mathbf{B}]$$

Leap-frog algorithm



Boris "exact" change of position

Implementation (leap-frog): first velocity estimation

[simplified here by neglecting Larmor orbit correction]

$$\begin{aligned}\mathbf{v}_{N+\frac{1}{2}} &= \mathbf{v}_{N-\frac{1}{2}} + \frac{Z_i e}{m}[\mathbf{E}_N + \mathbf{v}_N \times \mathbf{B}_N]\Delta t \\ &= \mathbf{v}_{N-\frac{1}{2}} + \frac{Z_i e}{m}\left[\mathbf{E}_N + \frac{\mathbf{v}_{N-\frac{1}{2}} + \mathbf{v}_{N+\frac{1}{2}}}{2} \times \mathbf{B}_N\right]\Delta t\end{aligned}$$

$$\mathbf{v}_{N+\frac{1}{2}} - \mathbf{v}_{N-\frac{1}{2}} = f_\epsilon\left[2\frac{\mathbf{E}}{B} + (\mathbf{v}_{N+\frac{1}{2}} + \mathbf{v}_{N-\frac{1}{2}}) \times \mathbf{b}\right]$$

$\Delta t/2 << 1/\omega_c$

~1/(2.1*10^8)

$$f_\epsilon = \frac{Z_i e B}{m}\frac{\Delta t}{2} = \omega_c \frac{\Delta t}{2}$$

$$\mathbf{v}_+ = \mathbf{v}_{N+\frac{1}{2}} - f_\epsilon \frac{\mathbf{E}}{B}$$

$$\mathbf{v}_- = \mathbf{v}_{N-\frac{1}{2}} + f_\epsilon \frac{\mathbf{E}}{B}$$

$$\mathbf{v}_+ - \mathbf{v}_- = f_\epsilon[(\mathbf{v}_+ + \mathbf{v}_-) \times \mathbf{b}]$$

$$\mathbf{b} = \frac{\mathbf{B}}{B}$$

The Boris integration algorithm

$$\mathbf{v}_{N+\frac{1}{2}}-\mathbf{v}_{N-\frac{1}{2}} = f_\epsilon\left[2\frac{\mathbf{E}}{B} + (\mathbf{v}_{N+\frac{1}{2}} + \mathbf{v}_{N-\frac{1}{2}}) \times \mathbf{b}\right] \qquad f_\epsilon = \frac{Z_i e B \Delta t}{m\ \ 2} = \omega_c\frac{\Delta t}{2} \qquad \mathbf{b} = \frac{\mathbf{B}}{B}$$

$$\mathbf{v}_+ = \mathbf{v}_{N+\frac{1}{2}} - f_\epsilon\frac{\mathbf{E}}{B}$$

$$\mathbf{v}_- = \mathbf{v}_{N-\frac{1}{2}} + f_\epsilon\frac{\mathbf{E}}{B}$$

$$\mathbf{v}_+ - \mathbf{v}_- = f_\epsilon\left[(\mathbf{v}_+ + \mathbf{v}_-) \times \mathbf{b}\right]$$

$$\left[\mathbb{I} - f_\epsilon \mathbb{X}_b\right]\mathbf{v}_+ = \left[\mathbb{I} + f_\epsilon \mathbb{X}_b\right]\mathbf{v}_-$$

$$\mathbf{v}_+ = \left[\mathbb{I} - f_\epsilon \mathbb{X}_b\right]^{-1}\left[\mathbb{I} + f_\epsilon \mathbb{X}_b\right]\mathbf{v}_-$$

Vector product matrix:

$$\mathbb{X}_b = \frac{1}{B}\begin{pmatrix} 0 & B_\varphi & -B_Z \\ B_\varphi & 0 & B_R \\ B_Z & -B_R & 0 \end{pmatrix}$$

$$[\mathbf{I} - \mathbf{R}\mathcal{U}]^{-1} = \frac{1}{\det(\mathbf{I} - \mathbf{R}\mathcal{U})}\begin{pmatrix} 1 + \mathcal{U}^2 b_x^2 & \mathcal{U} b_z + \mathcal{U}^2 b_x b_y & -\mathcal{U} b_y + \mathcal{U}^2 b_x b_z \\ -\mathcal{U} b_z + \mathcal{U}^2 b_x b_y & 1 + \mathcal{U}^2 b_y^2 & \mathcal{U} b_x + \mathcal{U}^2 b_y b_z \\ \mathcal{U} b_y + \mathcal{U}^2 b_x b_z & -\mathcal{U} b_x + \mathcal{U}^2 b_y b_z & 1 + \mathcal{U}^2 b_z^2 \end{pmatrix}$$

$$\mathbf{u} \otimes \mathbf{v} = \mathbf{u}\mathbf{v}^\top$$

$$(\mathbf{u} \otimes \mathbf{v})\mathbf{w} = (\mathbf{v} \cdot \mathbf{w})\mathbf{u}$$

$$\mathbf{b}\mathbf{b}^\top = (\mathbf{b} \cdot \mathbf{v})\,\mathbf{b}$$

The Boris integration algorithm

$$\mathbf{v}_{N+\frac{1}{2}} - \mathbf{v}_{N-\frac{1}{2}} = f_\epsilon \left[2\frac{\mathbf{E}}{B} + (\mathbf{v}_{N+\frac{1}{2}} + \mathbf{v}_{N-\frac{1}{2}}) \times \mathbf{b} \right] \qquad f_\epsilon = \frac{Z_i e B \Delta t}{m \ \ 2} = \omega_c \frac{\Delta t}{2} \qquad \mathbf{b} = \frac{\mathbf{B}}{B}$$

$$\mathbf{v}_+ = \mathbf{v}_{N+\frac{1}{2}} - f_\epsilon \frac{\mathbf{E}}{B}$$

$$\mathbf{v}_- = \mathbf{v}_{N-\frac{1}{2}} + f_\epsilon \frac{\mathbf{E}}{B}$$

$$\mathbf{v}_+ - \mathbf{v}_- = f_\epsilon \left[(\mathbf{v}_+ + \mathbf{v}_-) \times \mathbf{b} \right]$$

$$\left[\mathbb{I} - f_\epsilon \mathbb{X}_b \right] \mathbf{v}_+ = \left[\mathbb{I} + f_\epsilon \mathbb{X}_b \right] \mathbf{v}_-$$

$$\mathbf{v}_+ = \left[\mathbb{I} - f_\epsilon \mathbb{X}_b \right]^{-1} \left[\mathbb{I} + f_\epsilon \mathbb{X}_b \right] \mathbf{v}_-$$

$$\left[\mathbb{I} - f_\epsilon \mathbb{X}_b \right]^{-1} = \left[\mathbb{I} + f_\epsilon^2 \mathbf{b}\mathbf{b}^\top + f_\epsilon \mathbb{X}_b \right] / (1 + f_\epsilon^2)$$

$$\mathbf{v}_+ = \frac{1}{1 + f_\epsilon^2} \left[\mathbb{I} + f_\epsilon^2 \mathbf{b}\mathbf{b}^\top + f_\epsilon \mathbb{X}_b \right] \left[\mathbb{I} + f_\epsilon \mathbb{X}_b \right] \mathbf{v}_-$$

Vector product matrix:

$$\mathbb{X}_b = \frac{1}{B} \begin{pmatrix} 0 & B_\varphi & -B_Z \\ B_\varphi & 0 & B_R \\ B_Z & -B_R & 0 \end{pmatrix}$$

$$\mathbb{X}_b^2 = -\mathbb{I}$$

The Boris integration algorithm

$$\mathbf{v}_{N+\frac{1}{2}} - \mathbf{v}_{N-\frac{1}{2}} = f_\epsilon \left[2\frac{\mathbf{E}}{B} + (\mathbf{v}_{N+\frac{1}{2}} + \mathbf{v}_{N-\frac{1}{2}}) \times \mathbf{b} \right] \qquad f_\epsilon = \frac{Z_i e B \Delta t}{m \quad 2} = \omega_c \frac{\Delta t}{2} \qquad \mathbf{b} = \frac{\mathbf{B}}{B}$$

$$\mathbf{v}_+ = \mathbf{v}_{N+\frac{1}{2}} - f_\epsilon \frac{\mathbf{E}}{B}$$

$$\mathbf{v}_- = \mathbf{v}_{N-\frac{1}{2}} + f_\epsilon \frac{\mathbf{E}}{B}$$

$$\mathbf{v}_+ - \mathbf{v}_- = f_\epsilon \left[(\mathbf{v}_+ + \mathbf{v}_-) \times \mathbf{b} \right]$$

$$\mathbf{v}_+ = \frac{1}{1+f_\epsilon^2} \left[\mathbb{I} + f_\epsilon^2 \mathbf{b}\mathbf{b}^\top + f_\epsilon \mathbb{X}_b \right] \left[\mathbb{I} + f_\epsilon \mathbb{X}_b \right] \mathbf{v}_-$$

$$\mathbf{v}_+ = \frac{1}{1+f_\epsilon^2} \left[\mathbb{I} + 2f_\epsilon \mathbb{X}_b + 2f_\epsilon^2 \mathbf{b}\mathbf{b}^\top + f_\epsilon^2 \mathbb{X}_b^2 + f_\epsilon^3 \mathbf{b}\mathbf{b}^\top \mathbb{X}_b \right] \mathbf{v}_-$$

$$\mathbf{v}_+ = \frac{1}{1+f_\epsilon^2} \left[\mathbb{I} + 2f_\epsilon \mathbb{X}_b + 2f_\epsilon^2 \mathbf{b}\mathbf{b}^\top + (2f_\epsilon^2 - f_\epsilon^2) \mathbb{X}_b^2 + f_\epsilon^3 \mathbf{b}\mathbf{b}^\top \mathbb{X}_b \right] \mathbf{v}_-$$

$$\mathbf{v}_+ = \frac{1}{1+f_\epsilon^2} \left[(\mathbb{I} + f_\epsilon^2) + 2f_\epsilon \mathbb{X}_b + 2f_\epsilon^2 \mathbf{b}\mathbf{b}^\top - 2f_\epsilon^2 \mathbb{I} + f_\epsilon^3 \mathbf{b}\mathbf{b}^\top \mathbb{X}_b \right] \mathbf{v}_-$$

$$\mathbf{v}_+ \simeq \left[\mathbb{I} + \frac{2f_\epsilon}{1+f_\epsilon^2} \left(\mathbb{X}_b - f_\epsilon \mathbb{I} + f_\epsilon \mathbf{b}\mathbf{b}^\top \right) \right] \mathbf{v}_-$$

Vector product matrix:

$$\mathbb{X}_b = \frac{1}{B} \begin{pmatrix} 0 & B_\varphi & -B_Z \\ B_\varphi & 0 & B_R \\ B_Z & -B_R & 0 \end{pmatrix}$$

$$\mathbb{X}_b^2 = -\mathbb{I}$$

Neglecting terms of order f^3

The Boris integration algorithm

$$\mathbf{v}_{N+\frac{1}{2}}-\mathbf{v}_{N-\frac{1}{2}} = f_\epsilon\left[2\frac{\mathbf{E}}{B} + (\mathbf{v}_{N+\frac{1}{2}} + \mathbf{v}_{N-\frac{1}{2}})\times\mathbf{b}\right] \qquad f_\epsilon = \frac{Z_i e B\Delta t}{m\ \ 2} = \omega_c\frac{\Delta t}{2} \qquad \mathbf{b} = \frac{\mathbf{B}}{B}$$

$$\mathbf{v}_+ = \mathbf{v}_{N+\frac{1}{2}}-f_\epsilon\frac{\mathbf{E}}{B}$$

$$\mathbf{v}_- = \mathbf{v}_{N-\frac{1}{2}}+f_\epsilon\frac{\mathbf{E}}{B}$$

$$\mathbf{v}_+-\mathbf{v}_- = f_\epsilon\left[(\mathbf{v}_+ + \mathbf{v}_-)\times\mathbf{b}\right]$$

$$\mathbf{v}_+ \simeq \left[\mathbb{I} + \frac{2f_\epsilon}{1+f_\epsilon^2}\left(\mathbb{X}_b - f_\epsilon\mathbb{I} + f_\epsilon\mathbf{b}\mathbf{b}^\top\right)\right]\mathbf{v}_-$$

$$\mathbf{v}_+ \simeq \left[\mathbb{I} + \frac{2f_\epsilon}{1+f_\epsilon^2}\mathbb{M}\right]\mathbf{v}_-$$

$$\tilde{\mathbf{v}}_{N+\frac{1}{2}} \simeq \left[\mathbb{I} + \frac{2f_\epsilon}{1+f_\epsilon^2}\mathbb{M}\right](\mathbf{v}_{N-\frac{1}{2}}+f_\epsilon\mathbf{E}/B)+f_\epsilon\mathbf{E}/B$$

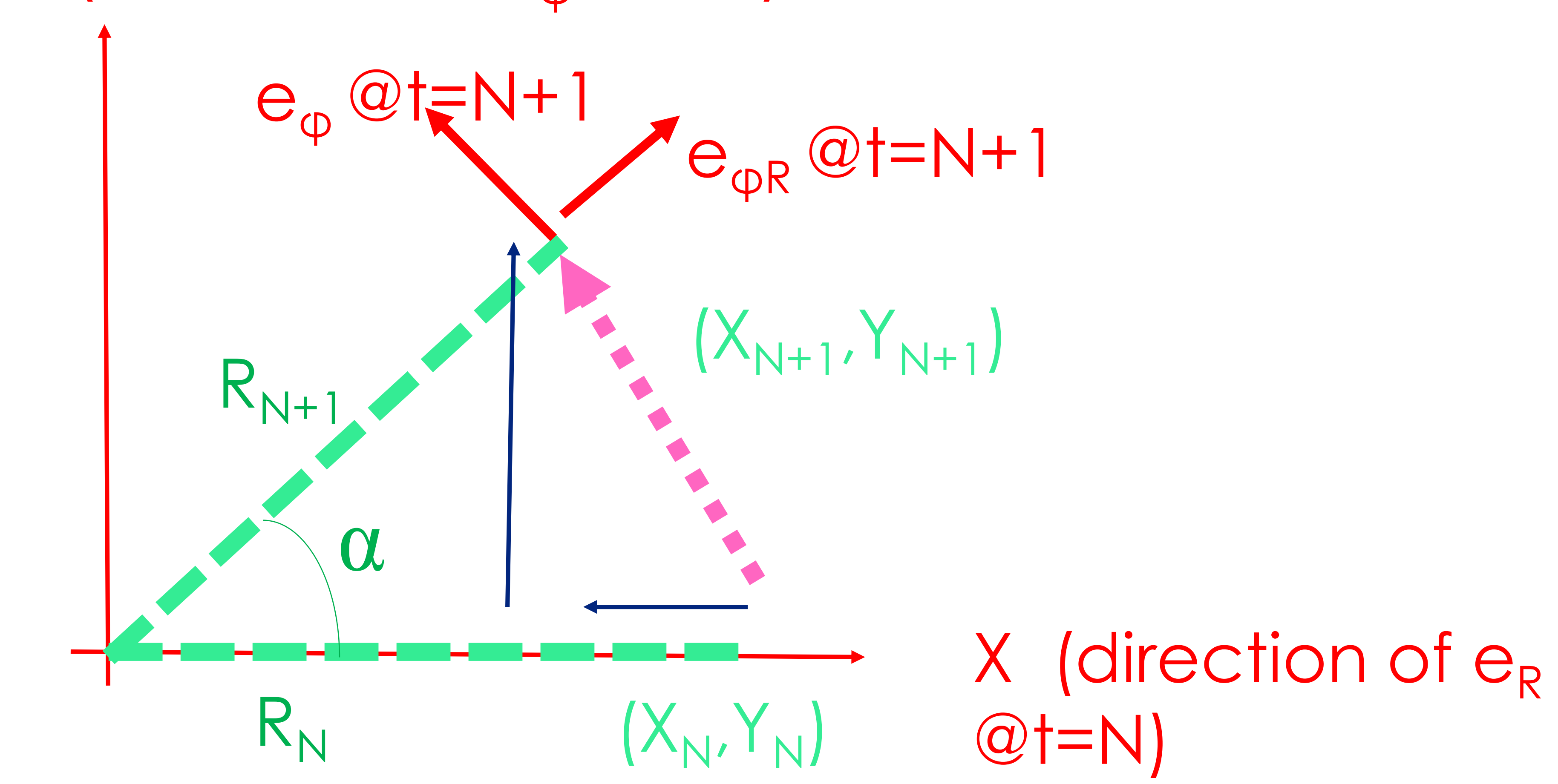
Vector product matrix:

$$\mathbb{X}_b = \frac{1}{B}\begin{pmatrix} 0 & B_\varphi & -B_Z \\ B_\varphi & 0 & B_R \\ B_Z & -B_R & 0 \end{pmatrix}$$

This is the evolution of velocity in the referential given by unit vectors at t=N

The Boris integration algorithm

Boris algorithm: **Evolving position**
In cylindrical coordinates means we need to move the coordinate system at each time step!



$$X_{N+1} = R_N + \tilde{v}_{X_{N+1/2}} \Delta t$$

$$Y_{N+1} = \tilde{v}_{\varphi_{N+1/2}} \Delta t$$

$$R_{N+1} = \sqrt{X_{N+1}^2 + Y_{N+1}^2}$$

$$\varphi_{N+1} = \varphi_N + \alpha$$

$$\alpha = \arcsin\left(\frac{Y_{N+1}}{R_{N+1}}\right)$$

$$v_{R_{N+1/2}} = \cos(\alpha)\,\tilde{v}_{R_{N+1/2}} + \sin(\alpha)\,\tilde{v}_{\varphi_{N+1/2}}$$

$$v_{\varphi_{N+1/2}} = -\sin(\alpha)\,\tilde{v}_{R_{N+1/2}} + \cos(\alpha)\,\tilde{v}_{\varphi_{N+1/2}}$$

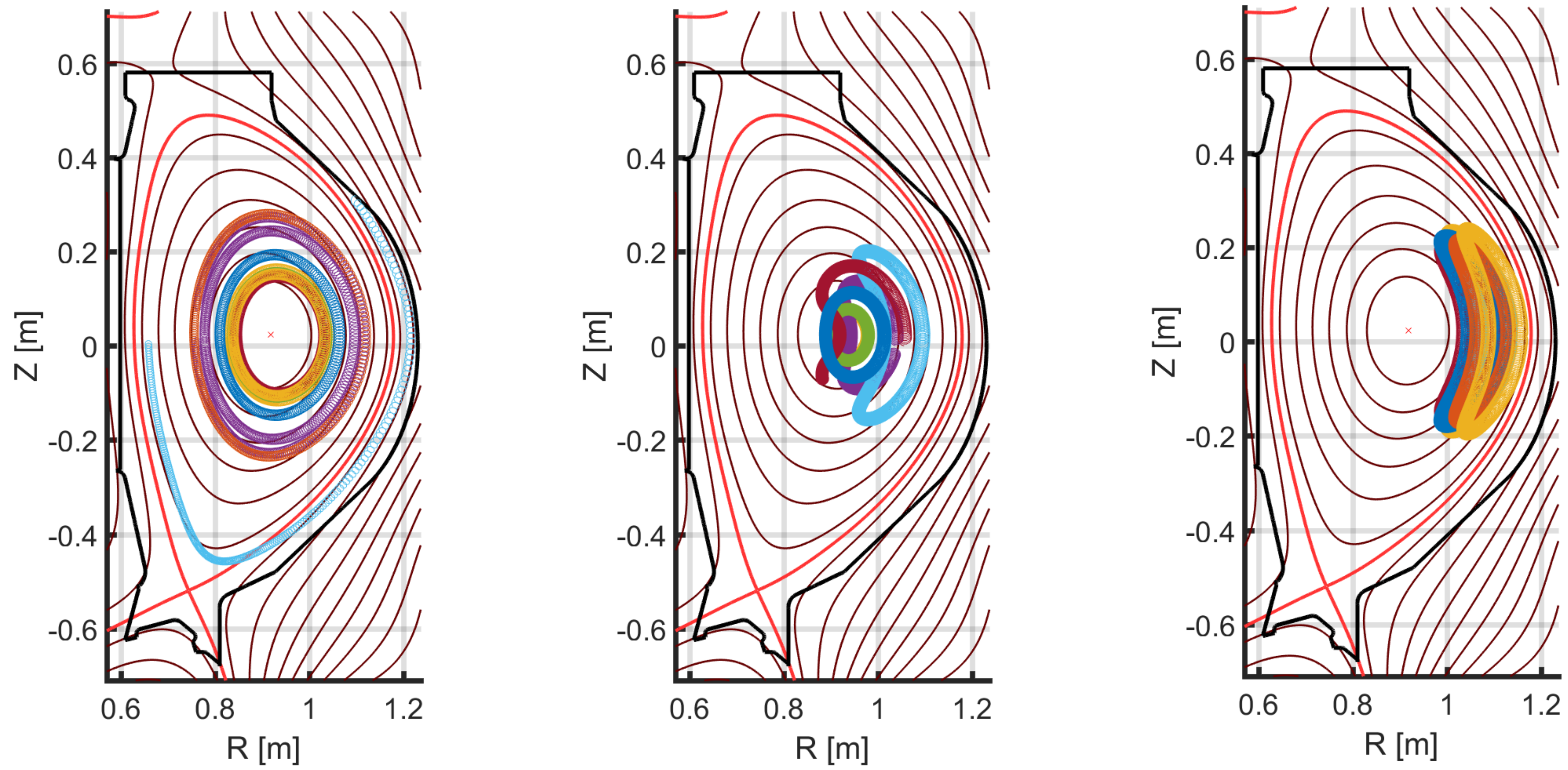
Rotation of angle α to express the vector velocity in the new coordinates

F. Jaulmes et al 2021 Nucl. Fusion 61 046012

Performances of integration scheme

Precision strongly depends on the quality of the underlying map for the poloidal flux ψ

When there are no collisions, 3D fields or Electric field, p_φ is conserved



Trajectories of 80keV ions deposited in the mid-plane with map ψ(R,Z) ~1200x2400

we define the canonical linear momentum as:

$$\mathbf{P} = m\mathbf{v} + Z_i e \mathbf{A},$$

$$\frac{1}{Z_i e}\frac{\mathrm{d}\mathbf{P}}{\mathrm{d}t} = \nabla(\mathbf{v}\cdot\mathbf{A} - \Phi)$$

$$\frac{1}{Z_i e}\frac{\mathrm{d}p_\varphi}{\mathrm{d}t} = \frac{\partial A_R}{\partial \varphi}v_R + \frac{\partial A_Z}{\partial \varphi}v_Z + \frac{\partial A_\varphi}{\partial \varphi}v_\varphi - \frac{\partial \Phi}{\partial \varphi}.$$

$$p_\varphi = m_f R v_\varphi - (Z_f e)(\psi_0 + \psi_1)$$

p_φ is the toroidal canonical angular momentum

F. Jaulmes et al 2021 Nucl. Fusion 61 046012

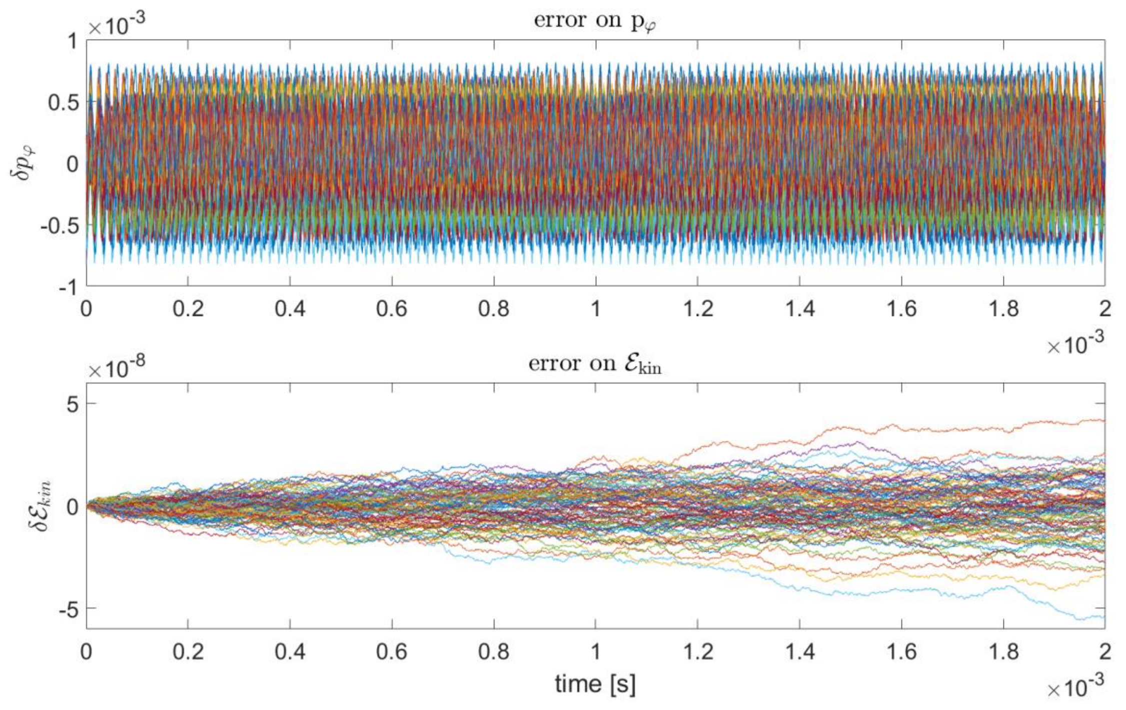
Performances of integration scheme

Precision strongly depends on the quality of the underlying map for the poloidal flux ψ

Of course decreasing time step size can influence the precision, down to some value when the grid precision dominates

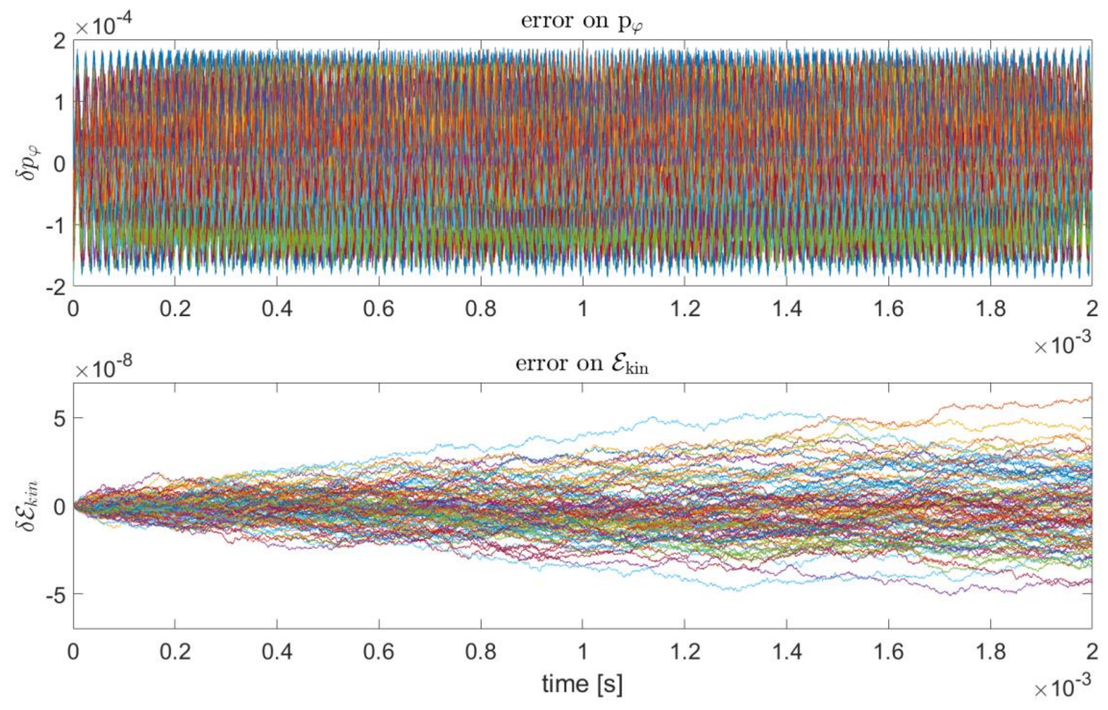
2 ms simulations of collisionless trajectories [80 keV ions]

$\delta t = 10^{-9}$ s



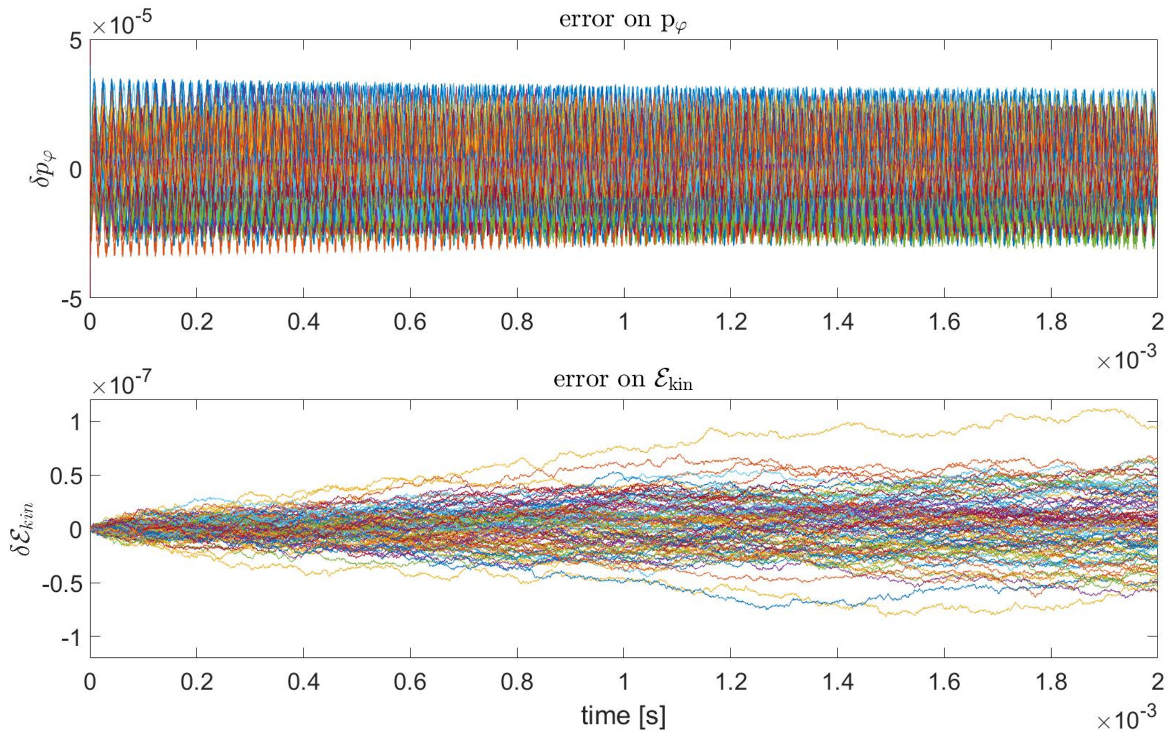
$\delta p_\varphi = 8*10^{-4}$
$\delta E_{kin} = 4*10^{-8}$ eV

$\delta t = 0.5\ 10^{-9}$ s



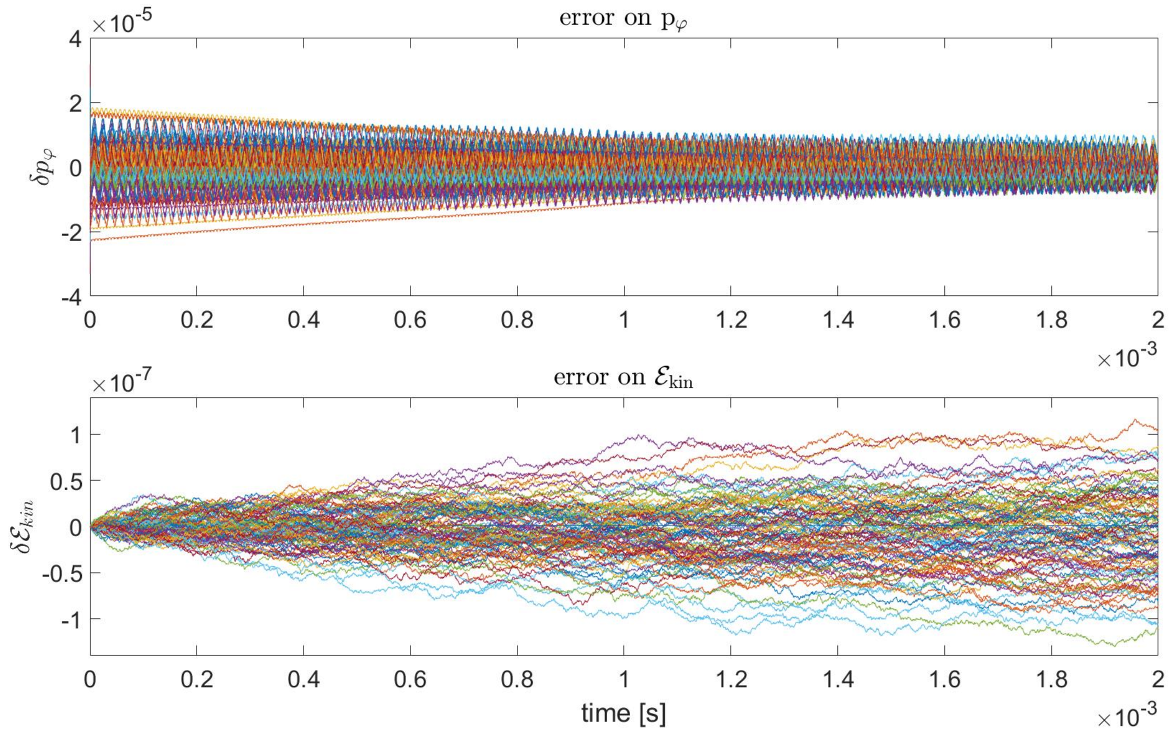
$\delta p_\varphi = 2*10^{-4}$
$\delta E_{kin} = 7*10^{-8}$ eV

$\delta t = 0.2\ 10^{-9}$ s



$\delta p_\varphi = 4*10^{-5}$
$\delta E_{kin} = 11*10^{-8}$ eV

$\delta t = 0.1\ 10^{-9}$ s



Cumulated error of ~10^{-5} / ms on p_φ

$\delta p_\varphi = 2*10^{-5}$
$\delta E_{kin} = 14*10^{-8}$ eV

The Boris integration algorithm

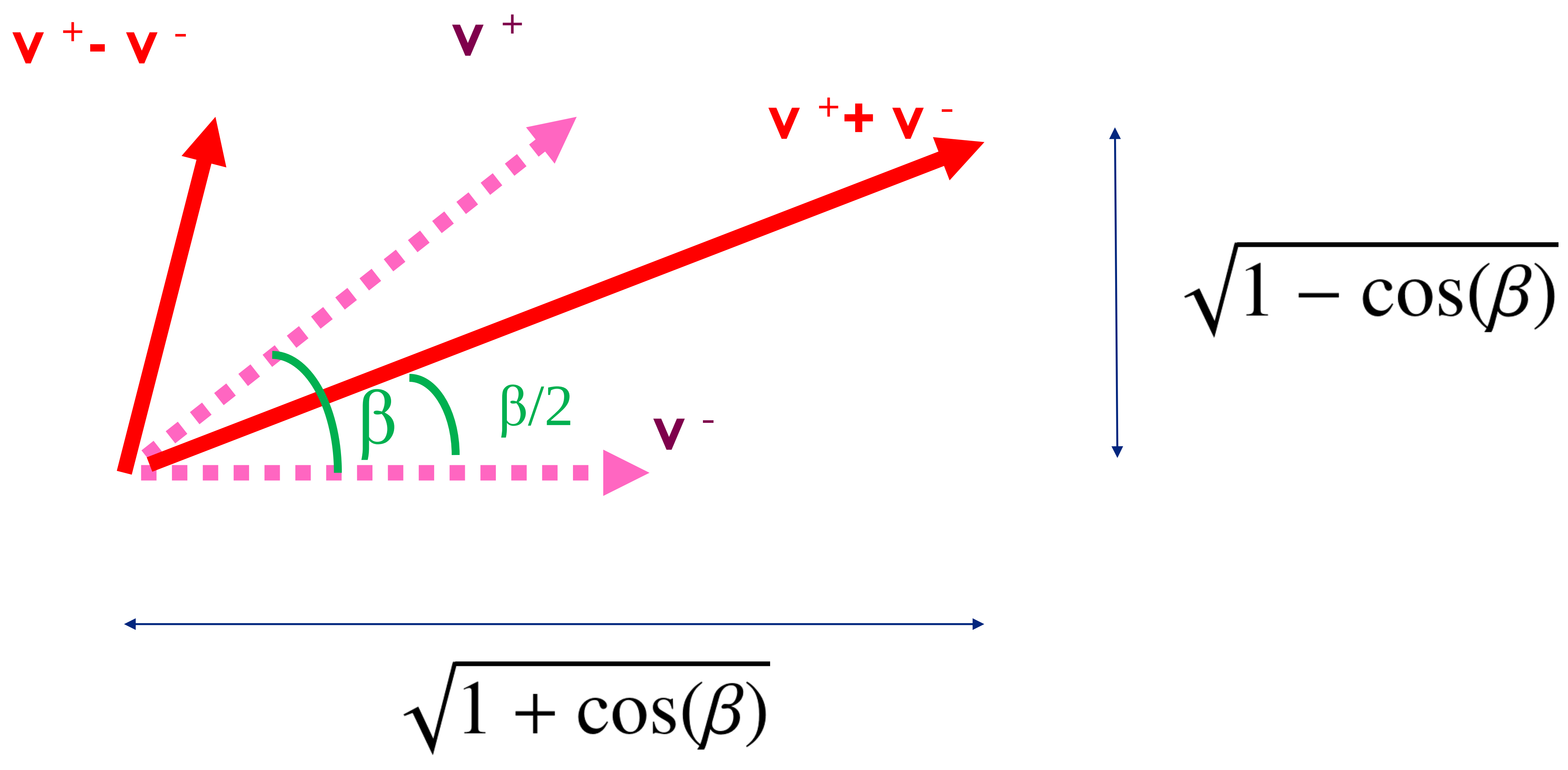
Boris algorithm: **Adjusting time step to match the cyclotron frequency [optional]**

Inaccuracy arising on long simulations due to the Larmor orbit?

$$\mathbf{v}_+ - \mathbf{v}_- = f_\epsilon \left[(\mathbf{v}_+ + \mathbf{v}_-) \times \mathbf{b} \right]$$

This rotation β is given by the equation:

$$\frac{|\mathbf{v}_+^\perp - \mathbf{v}_-^\perp|}{|\mathbf{v}_+^\perp - \mathbf{v}_-^\perp|} = |\tan(\beta/2)| = f_\epsilon$$



S. Parker and C. Birdsall, "Numerical error in electron orbits with large $\omega_c\Delta t$" Journal of Computational Physics, vol. 97, 91-102 (1991). http://linkinghub.elsevier.com/retrieve/pii/002199919190040R

β *should* be $\omega_c \Delta t$, ----> $f_\epsilon = \tan\left(\frac{Z_i e B}{m} \frac{\Delta t}{2}\right) = \tan\left(\omega_c \frac{\Delta t}{2}\right)$

F. Jaulmes et al 2021 Nucl. Fusion 61 046012

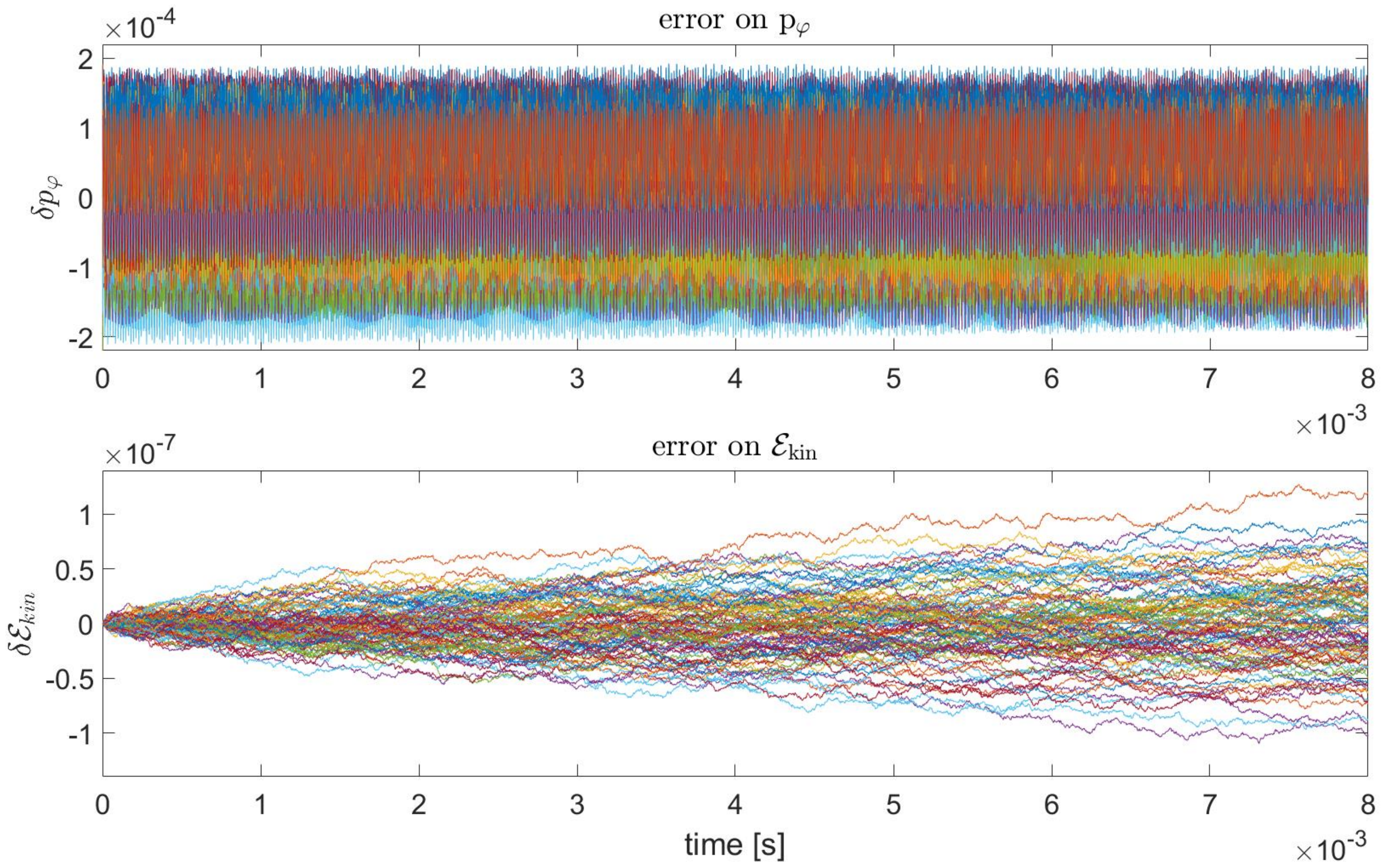
The Boris integration algorithm

Boris algorithm: **Adjusting time step to match the cyclotron frequency** **[optional]**

$$f_\epsilon = \frac{Z_i e B}{m}\frac{\Delta t}{2} = \omega_c \frac{\Delta t}{2} \dashrightarrow f_\epsilon = \tan\left(\frac{Z_i e B}{m}\frac{\Delta t}{2}\right) = \tan\left(\omega_c \frac{\Delta t}{2}\right)$$

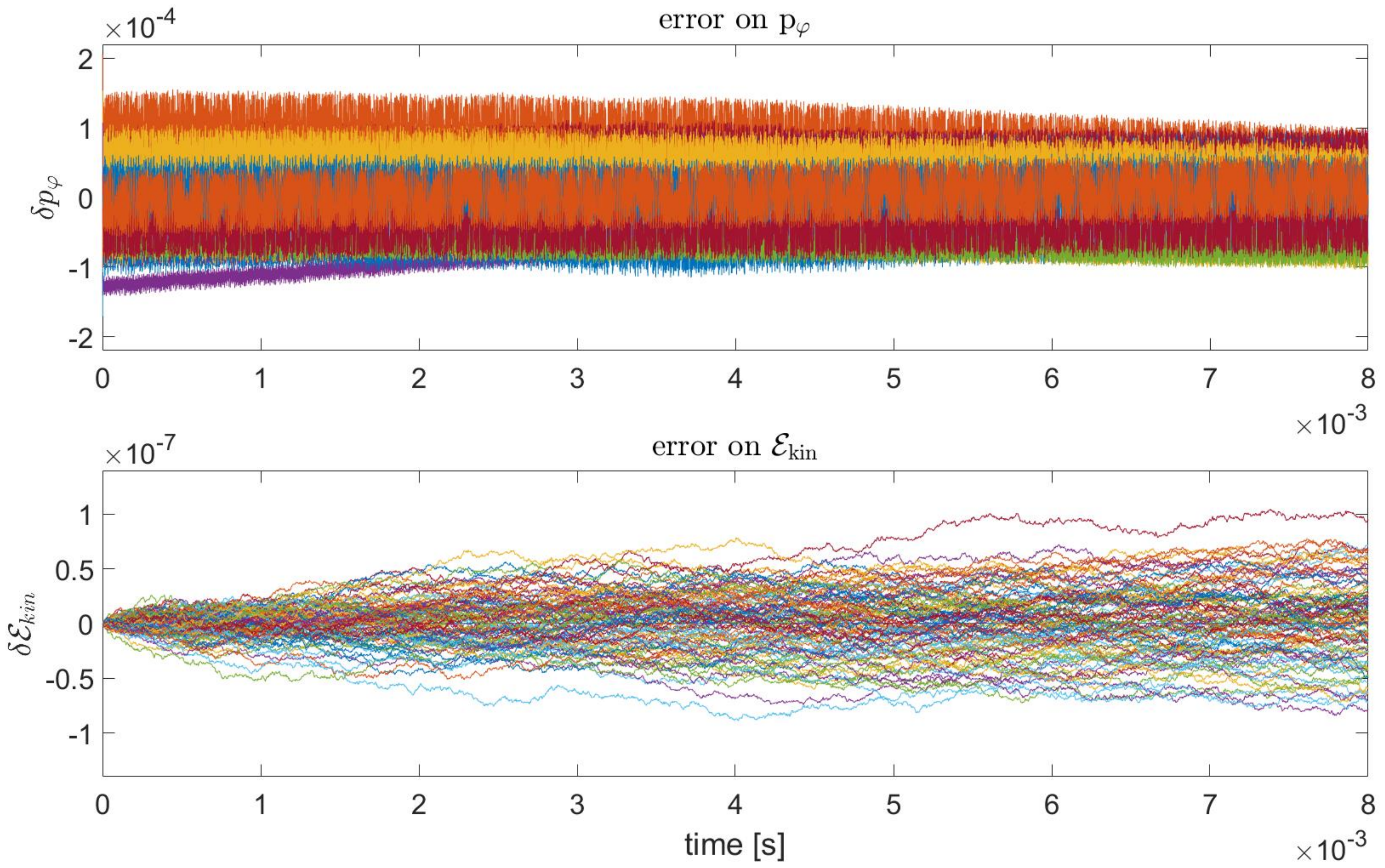
δt = 0.5 10^{-9} s

With Larmor orbit correction



Larger periodic excursions on p_φ

Without Larmor orbit correction



Cumulated error appears on p_φ

F. Jaulmes et al 2021 Nucl. Fusion 61 046012

Overview of this lecture

- What is the NBI [Neutral Beam Injector]
- Modelling particle orbits in tokamaks
- Coulomb Collisions and slowing down distribution
- Application: modelling of fast neutrons generation in COMPASS Upgrade

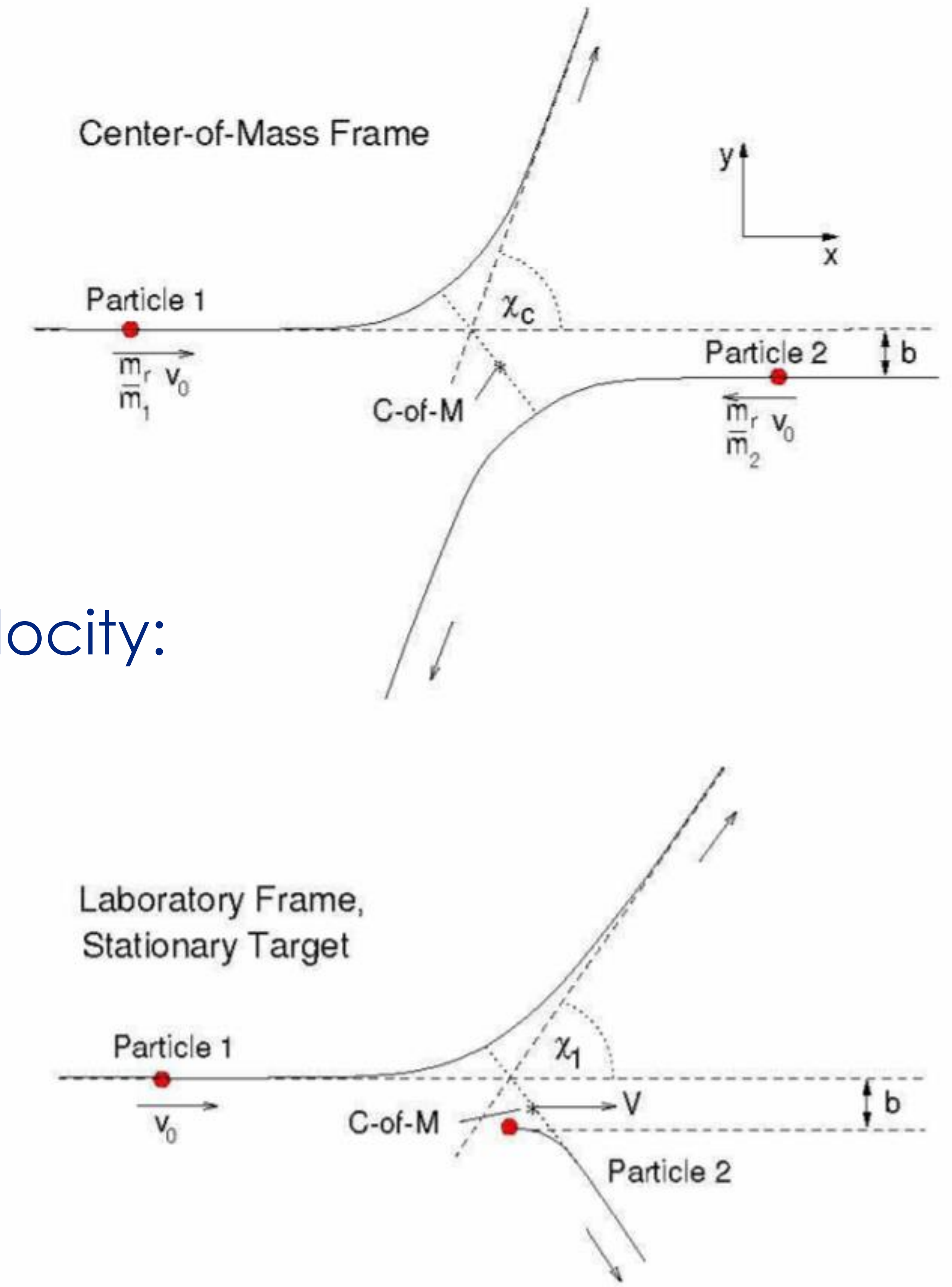
Coulomb collisions: principles

Collision:

$$m_r = \frac{m_1 m_2}{m_1 + m_2}$$

Center of mass frame velocity:

$$\mathbf{V} = \frac{m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2}{m_1 + m_2}$$



Fast ion collisions: NBI D ion against thermal electrons and ions (D):

[PP&FE – J. Freidberg pp. 201-203]

$$\frac{dp_{x_1}}{dt} = -\nu_{12} p_{x_1} \qquad \frac{d\mathcal{E}_1}{dt} = -\nu_{12} \frac{2m_1}{m_1 + m_2} \mathcal{E}_1$$

time step = 10^{-9} s
time simulated = 20 ms ($2\tau_s$)
Collision time step = 10^{-8} s ($2\tau_{ci}$)

$$\frac{dv_1}{dt} = -\nu_{12} \frac{m_1}{m_1 + m_2} v_1$$

See : https://ocw.mit.edu/courses/nuclear-engineering/22-611j-introduction-to-plasma-physics-i-fall-2006/readings/chap3.pdf

Overall loss: fast beam D against background e and i

$$\nu_{be} \simeq \left(\frac{1}{4\pi}\frac{e^4 n_e}{\epsilon_0^2 m_D m_e}\ln\Lambda\right)\frac{1}{v_b^3 + 1.33 v_{\mathrm{th_e}}^3}$$

$$\nu_{bi} \simeq \left(\frac{1}{2\pi}\frac{e^4 n_i}{\epsilon_0^2 m_D^2}\ln\Lambda\right)\frac{1}{v_b^3 + 1.33 v_{\mathrm{th_i}}^3}$$

momentum

$$\nu_{be} \simeq \left(\frac{1}{4\pi}\frac{e^4 n_e}{\epsilon_0^2 m_D m_e}\ln\Lambda\right)\frac{1}{v_b^3 + 1.33 v_{\mathrm{th_e}}^3}$$

$$\nu_{bi} \simeq \left(\frac{1}{4\pi}\frac{e^4 n_i}{\epsilon_0^2 m_D^2}\ln\Lambda\right)\frac{1}{v_b^3 + 1.33 v_{\mathrm{th_i}}^3}$$

norm of velocity

Equaling loss rates on e and i:

$$v_c^3 \simeq 1.33\frac{n_i}{n_e}\frac{m_e}{m_D}v_{\mathrm{th}_e}^3$$

$$\mathcal{E}_c \simeq \left(1.33\frac{n_i}{n_e}\right)^{2/3}\left(\frac{m_D}{m_e}\right)^{1/3} T_e \simeq 18.66\, T_e$$

→ Over-simplified view of things: we need to account for

- random distributions
- pitch angle scattering

Stochastization of the losses: variance in velocity norm

Fokker-Planck equation: collision term

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{x}} + \frac{e_J}{m_J}(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \cdot \frac{\partial f}{\partial \boldsymbol{v}} = \left(\frac{\partial f}{\partial t}\right)_c$$

Tokamaks, 3rd edition
J. Wesson

$$\left(\frac{\partial f}{\partial f}\right)_c = \frac{f(\boldsymbol{x}, \boldsymbol{v}, t + \Delta t) - f(\boldsymbol{x}, \boldsymbol{v}, t)}{\Delta t}.$$

$$\left(\frac{\partial f}{\partial t}\right)_c = -\sum_\alpha \frac{\partial}{\partial v_\alpha}(\langle \Delta v_\alpha \rangle f) + \frac{1}{2}\sum_{\alpha,\beta} \frac{\partial^2}{\partial v_\alpha \partial v_\beta}(\langle \Delta v_\alpha \Delta v_\beta \rangle f).$$

The quantity $\langle \Delta v_\alpha \rangle$ is called the coefficient of dynamic friction and $\langle \Delta v_\alpha \Delta v_\beta \rangle$ the diffusion tensor.

Averaging on gyrophase Φ

The *drift kinetic equation* is an equation for the gyro-averaged distribution function

$$\bar{f} = \frac{1}{2\pi} \int f \, d\phi,$$

$$\frac{\partial \bar{f}}{\partial t} + \boldsymbol{v}_g \cdot \nabla \bar{f} + \left[e_J \boldsymbol{E} \cdot \boldsymbol{v}_g + \mu \frac{\partial B}{\partial t} \right] \frac{\partial \bar{f}}{\partial K} = \left(\frac{\partial \bar{f}}{\partial t}\right)_c,$$

$$\boldsymbol{v}_g = v_\parallel \boldsymbol{b} + \frac{\boldsymbol{E} \times \boldsymbol{B}}{B^2} + \frac{v_\parallel^2 \boldsymbol{b} \times (\boldsymbol{b} \cdot \nabla)\boldsymbol{b} + \mu \boldsymbol{b} \times \nabla B}{\omega_{cJ}}$$

where $\boldsymbol{b} = \boldsymbol{B}/|\boldsymbol{B}|$ and $\omega_{cJ} = e_J B / m_J$.

$$\left(\frac{\partial f}{\partial t}\right)_c = \sum_J \frac{e^2 Z^2 Z_J^2 \ln \Lambda}{8\pi \varepsilon_0^2 m} \times \frac{\partial}{\partial v_\alpha} \int \left(\frac{f_J(\boldsymbol{v}_J)}{m} \frac{\partial f(\boldsymbol{v})}{\partial v_\beta} - \frac{f(\boldsymbol{v})}{m_J} \frac{\partial f_J(\boldsymbol{v}_J)}{\partial v_{J\beta}} \right) u_{\alpha\beta} \, d\boldsymbol{v}_J$$

where

$$\boldsymbol{u} = \boldsymbol{v} - \boldsymbol{v}_J \quad \text{and} \quad u_{\alpha\beta} = \frac{u^2 \delta_{\alpha\beta} - u_\alpha u_\beta}{u^3}.$$

Stochastization of the losses: variance in velocity norm

We admit: Fokker Plank equation for fast ion $v_i<v<v_e$

$$\left.\frac{\partial f}{\partial t}\right|_c = \frac{1}{\tau_s v^2}\frac{\partial}{\partial v}(v^3+v_c^3)f + \frac{1}{\tau_s v^2}\frac{\partial}{\partial v}\left(\frac{v^2 T_e}{m_b}+\frac{v_c^3 T_i}{m_b v}\right)\frac{\partial f}{\partial v} + \frac{\nu_{ii}}{2}\frac{\partial}{\partial \zeta}(1-\zeta^2)\frac{\partial}{\partial \zeta}f,$$

$$= \frac{1}{\tau_s v^2}\frac{\partial}{\partial v}[(v^3+v_c^3)f] + \frac{\beta}{\tau_s}\frac{v_c^3}{v^3}\frac{\partial}{\partial \xi}(1-\xi^2)\frac{\partial f}{\partial \xi} + \frac{1}{\tau_s v^2}\frac{\partial}{\partial v}\left[\left(\frac{T_e}{m_{fi}}v^2+\frac{T_i}{m_{fi}}\frac{v_c^3}{v}\right)\frac{\partial f}{\partial v}\right]$$

$$\tau_s = 6.32\cdot 10^8\cdot\frac{A_{fi}}{Z_{fi}^2\ln\Lambda_e}\cdot\frac{(T_e\,[\mathrm{eV}])^{3/2}}{n_e\,[\mathrm{cm}^{-3}]}\ \mathrm{s} \qquad \tau_s = (1/\nu_{be})_{v=0}$$

$$v_c = 5.33\cdot 10^4\cdot\sqrt{T_e\,[\mathrm{eV}]}\cdot\left\langle\frac{Z_i^2}{A_i}\right\rangle^{1/3}\ \mathrm{m/s}$$

$$\beta = \frac{\langle Z_i^2\rangle}{2\left\langle\frac{Z_i^2}{A_i}\right\rangle A_{fi}},\quad \left\langle\frac{Z_i^2}{A_i}\right\rangle = \frac{\sum_i n_i(Z_i^2/A_i)\ln\Lambda_i}{n_e\ln\Lambda_e},\quad \langle Z_i^2\rangle = \frac{\sum_i n_i Z_i^2\ln\Lambda_i}{n_e\ln\Lambda_e}$$

Taking $f=\delta(v-v_0)\,\delta(\zeta-\zeta_0)$,

→

$$v' = v_0 - \delta t\left[\frac{v_0}{\tau_s}(1-2T_e/m_b v_0^2) + \frac{v_c^3}{v_0^2\tau_s}(1+T_i/m_b v_0^2)\right],$$

Evolution of norm of velocity during δt

New Techniques for Calculating Heat and Particle Source Rates due to Neutral Beam Injection in Axisymmetric Tokamaks,
R.J. Goldston et al., J. Comp. Phys. 43 (1981) 61

Stochastization of the losses: variance in velocity norm

Fokker Plank equation for fast ion $v_i<v<v_e$

New Techniques for Calculating Heat and Particle Source Rates due to Neutral Beam Injection in Axisymmetric Tokamaks,
R.J. Goldston et al., J. Comp. Phys. 43 (1981) 61

$$v' = v_0 - \delta t \left[\frac{v_0}{\tau_s}(1 - 2T_e/m_b v_0^2) + \frac{v_c^3}{v_0^2 \tau_s}(1 + T_i/m_b v_0^2) \right],$$

$$\langle \Delta v^2 \rangle = \langle (v' - v)^2 \rangle = \langle (v')^2 + v^2 - 2vv' \rangle = \langle (v')^2 + v^2 - 2v(v + \delta t \, \langle \partial v/\partial t \rangle) \rangle$$

$$\langle \Delta v^2 \rangle = \langle (v')^2 - v^2 - 2v(\delta t \, \langle \partial v/\partial t \rangle) \rangle$$

$$\langle \Delta v^2 \rangle = \delta t \left(\langle \partial v^2/\partial t \rangle - 2v \, \langle \partial v/\partial t \rangle \right)$$

→ Taking $f = \delta(v - v_0)\,\delta(\zeta - \zeta_0)$, we arrive at

$$\langle \partial v/\partial t \rangle = -\frac{v}{\tau_s}(1 - 2T_e/m_b v^2) - \frac{v_c^3}{v^2 \tau_s}(1 + T_i/m_b v^2),$$

$$\langle \partial v^2/\partial t \rangle = \frac{\int (\partial f/\partial t)_C v^4 \mathrm{d}v \mathrm{d}\xi}{\int f v^2 \mathrm{d}v \mathrm{d}\xi} \quad \text{and} \quad \langle \partial v/\partial t \rangle = \frac{\int (\partial f/\partial t)_C v^3 \mathrm{d}v \mathrm{d}\xi}{\int f v^2 \mathrm{d}v \mathrm{d}\xi}$$

→ $$\langle \Delta v^2 \rangle = \frac{2\Delta t}{\tau_s} \left(\frac{T_e}{m_D} + \frac{v_C^3}{v^3} \frac{T_i}{m_D} \right) \qquad \tau_s = (1/\nu_{be})_{v=0}$$

$$\left.\frac{\partial f}{\partial t}\right|_c = \frac{1}{\tau_s v^2} \frac{\partial}{\partial v} (v^3 + v_c^3) f + \frac{1}{\tau_s v^2} \frac{\partial}{\partial v} \left(\frac{v^2 T_e}{m_b} + \frac{v_c^3 T_i}{m_b v} \right) \frac{\partial f}{\partial v}$$

The standard deviation will scale as square root of this.
For consistency, we use the critical energy related to "norm of velocity" loss rates, not momentum loss rates.

Pitch angle scattering

Diffusion in the perpendicular plane (often called σ_{90}or σ) :

See: https://ocw.mit.edu/courses/nuclear-engineering/22-611j-introduction-to-plasma-physics-i-fall-2006/readings/chap3.pdf

$$\sigma_{12} = \frac{2m_2}{m_1 + m_2}\nu_{12}$$

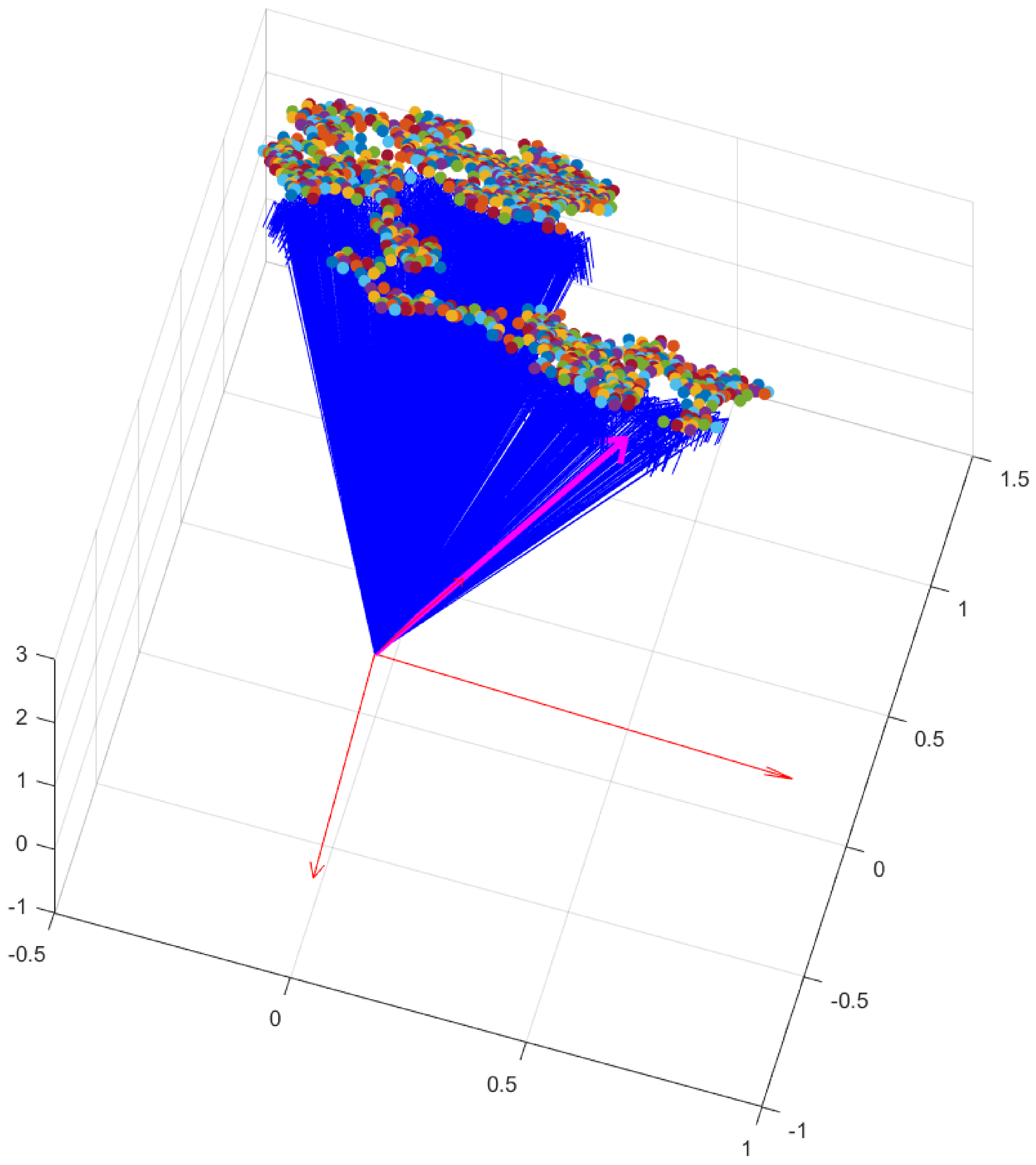
$$\sigma_{be} = \frac{2m_e}{m_D + m_e}\nu_{be} \simeq 0 \qquad \sigma_{bi} = \frac{2m_D}{m_D + m_D}\nu_{bi} = \nu_{bi}$$

Close value

See: F. Jaulmes et al 2021 Nucl. Fusion 61 046012

$$\delta\Omega = \mathcal{N}(1,1)\frac{\pi}{2}\sqrt{(\delta t)\nu_{\perp i}}$$

Random walk process



J.D. Callen. Draft Material for Fundamentals of Plasma Physics book, 2006. homepages.cae.wisc.edu/~callen/chap2.pdf.

Full-orbit simulations with the EBdyna code allow to obtain description of NBI in space and velocity space

F. Jaulmes et al 2021 Nucl. Fusion 61 046012

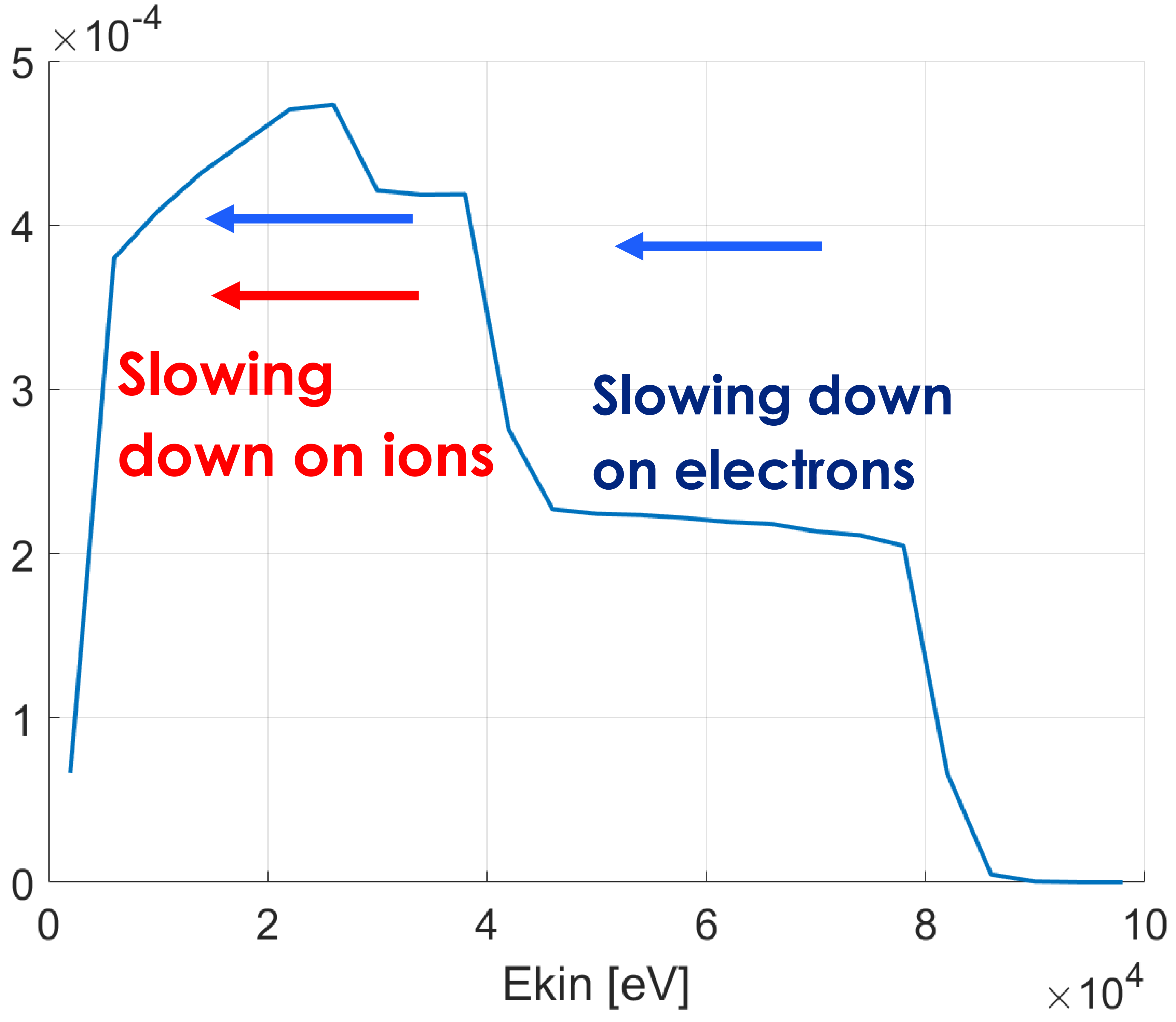
Scenario	Bt [T]	Ip [MA]	n_e [10^{20} m^{-3}]	τ_e [ms]	Ecrit [keV]	$n_{0,wall}$ [10^{18} m^{-3}]
#24300	4.3	1.2	1.9	83	40	2.6

Slowing down distribution of the 80 keV NBI

$$\tau_s = \frac{m_f}{m_e}\frac{1}{3\nu_e}\ln\left[1+\left(\frac{\mathcal{E}_0}{\mathcal{E}_{crit}}\right)^{3/2}\right] \text{ with } \mathcal{E}_{crit} \simeq 18.6 T_e$$

F. Jaulmes et al 2021 Nucl. Fusion 61 046012

EBdyna CU24300 for 1MW NBI @R_f=0.6m



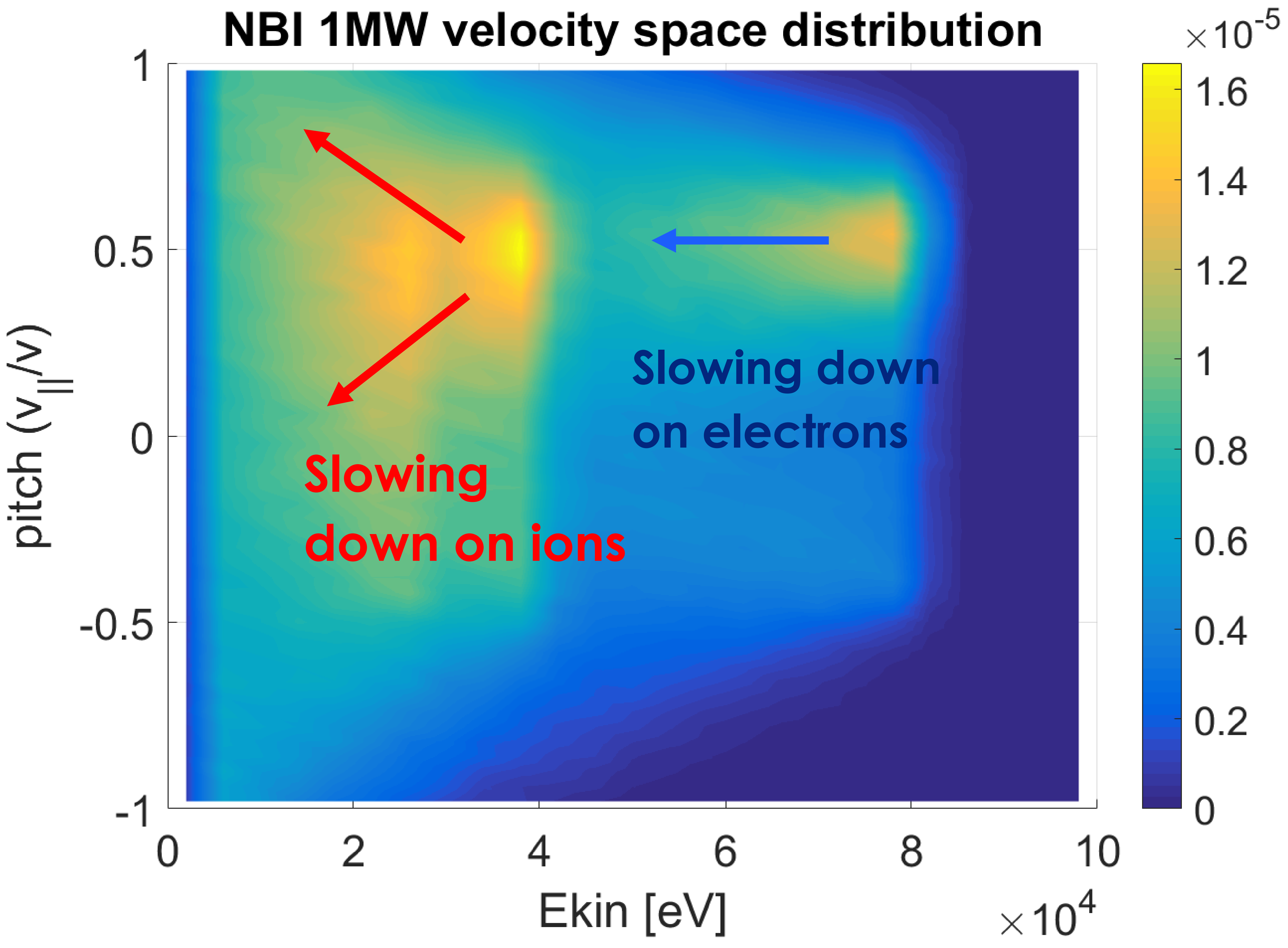
Full-orbit simulations with the EBdyna code allow to obtain description of NBI in space and velocity space

F. Jaulmes et al 2021 Nucl. Fusion 61 046012

Scenario	Bt [T]	Ip [MA]	n_e [10^{20} m^{-3}]	τ_e [ms]	Ecrit [keV]	$n_{0,wall}$ [10^{18} m^{-3}]
#24300	4.3	1.2	1.9	83	40	2.6

$$\delta\Omega = \mathcal{N}(1,1)\frac{\pi}{2}\sqrt{(\delta t)\nu_{\perp i}}$$

EBdyna CU24300 for 1MW NBI @R_t=0.6m

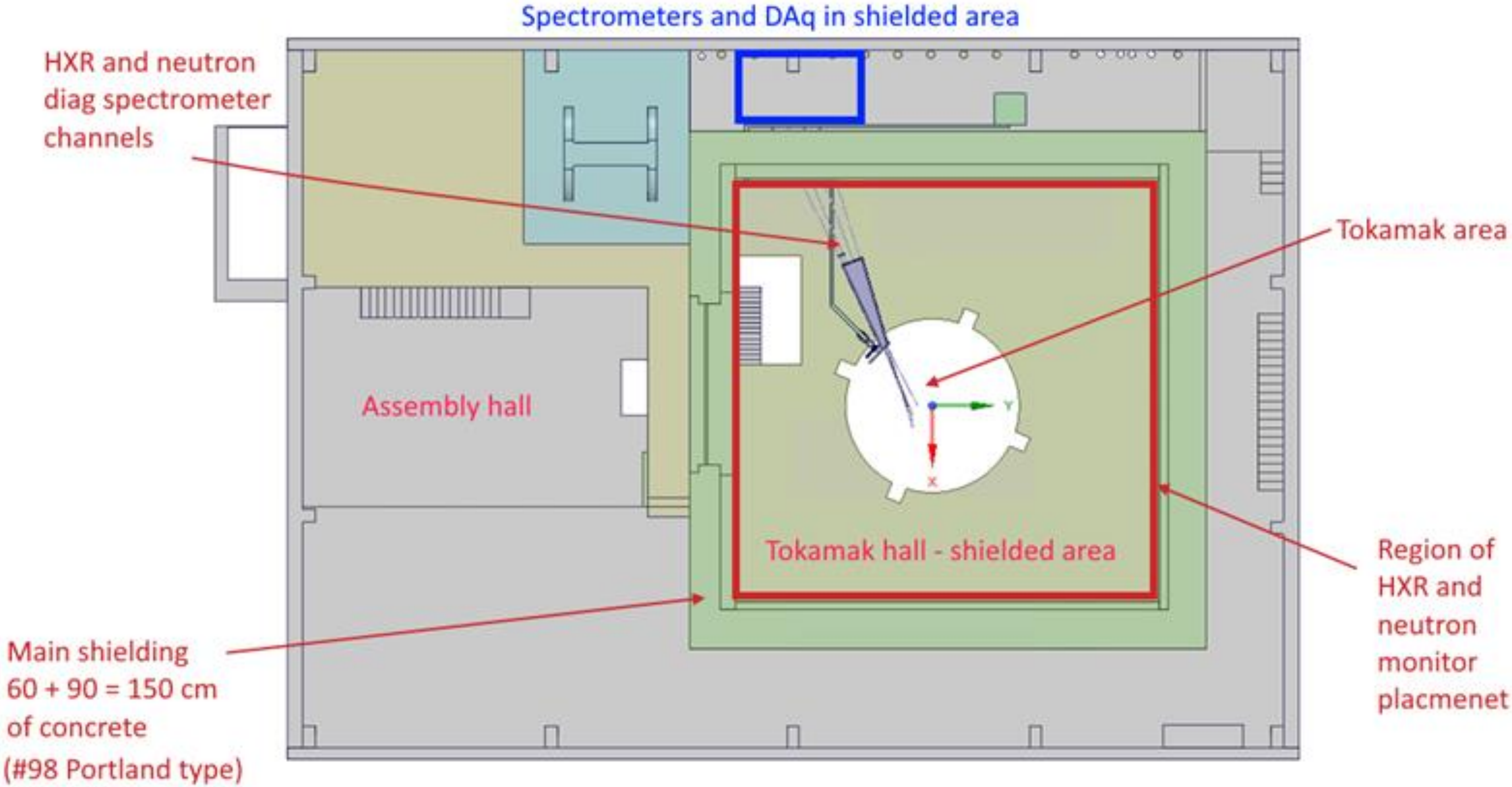


Overview of this lecture

- What is the NBI [Neutral Beam Injector]
- Modelling particle orbits in tokamaks
- Coulomb Collisions and slowing down distribution
- **Application: modelling of fast neutrons generation in COMPASS Upgrade**

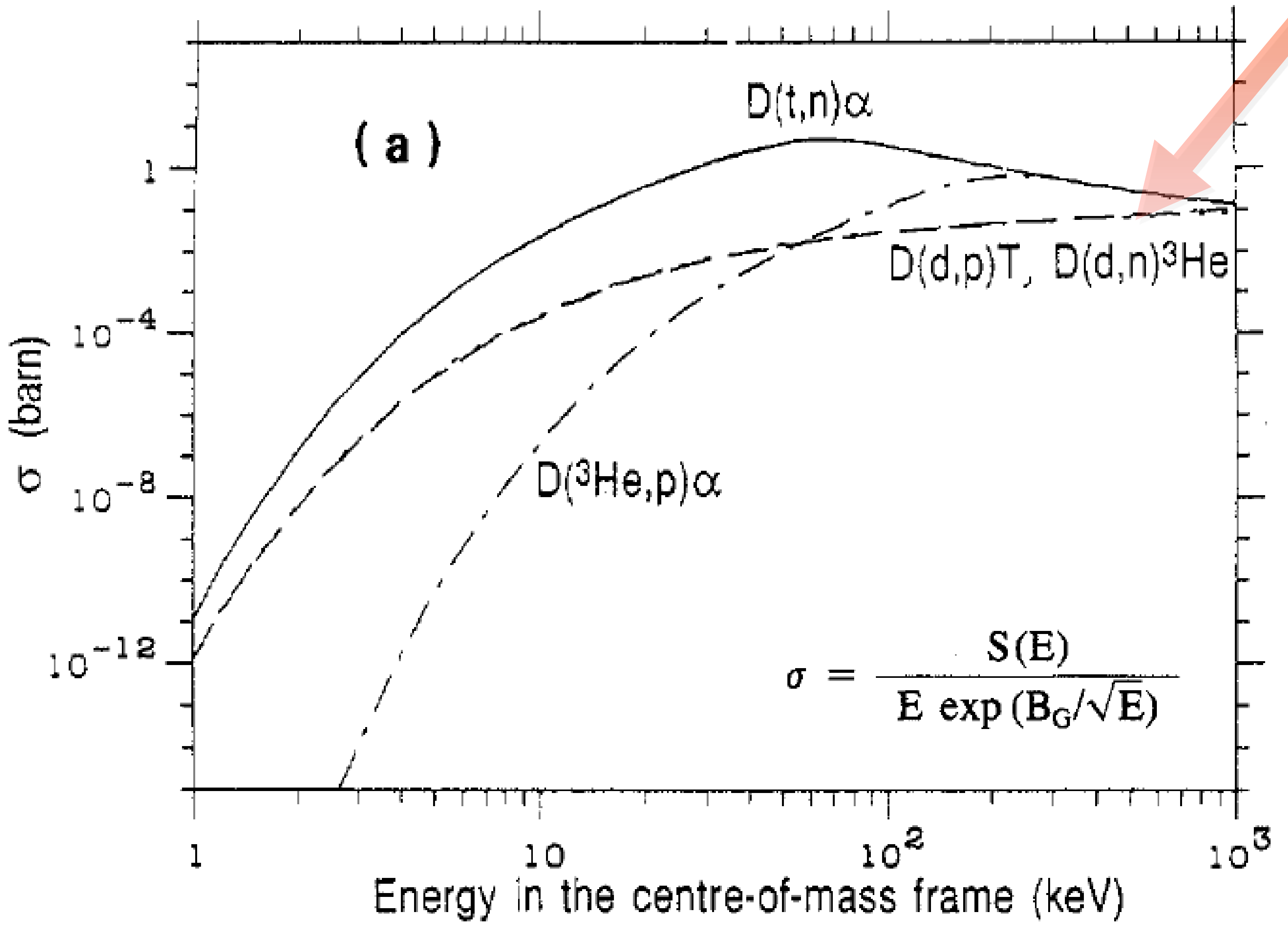
F. Jaulmes et al.: Journal of Fusion Energy volume 41, Article number: 16 (2022)

NEUTRON DIAGNOSTICS IN THE EXPERIMENTAL HALL



NEUTRON YIELD: BEAM -> TARGET (H.-S. BOSCH AND G.M. HALE 1992 NUCL. FUSION 32 611)

Throughout this paper, E denotes the energy available in the CM frame. For a particle A with mass m_A striking a stationary particle B, the simple relation $E_A = E\,(m_A + m_B)/m_B$ holds.



(2i) $^2_1D + ^2_1D \rightarrow ^3_1T$ (1.01 MeV) + p^+ (3.02 MeV)

(2ii) $\rightarrow ^3_2He$ (0.82 MeV) + n^0 (2.45 MeV)

$$dR/dV = \frac{n_i n_j}{1 + \delta_{ij}} \langle \sigma v \rangle \qquad (10)$$

where n_i, n_j are the particle densities and δ_{ij} is the Kronecker symbol. With $f(\vec{v}_i)$ the velocity distribution of a particle and g the relative velocity ($\vec{g} = \vec{v}_i - \vec{v}_j$), we obtain

$$\langle \sigma v \rangle = \int\int f(\vec{v}_i)\, f(\vec{v}_j)\, \sigma(|g|)\, |g|\, d\vec{v}_i\, d\vec{v}_j \qquad (11)$$

$$\sigma = \frac{S(\mathcal{E}_{com})}{\exp(B_g/\sqrt{\mathcal{E}_{com}})} \text{ in mb } (10^{-28}\text{m}^2) \text{ with } B_g = 31.3970\sqrt{\text{keV}}$$

NEUTRON YIELD:

(2i) ${}^2_1D + {}^2_1D \rightarrow {}^3_1T$ (1.01 MeV) + p^+ (3.02 MeV)

(2ii) $\rightarrow {}^3_2He$ (0.82 MeV) + n^0 (2.45 MeV)

$$\sigma = \frac{S(\mathcal{E}_{\rm com})}{\exp(B_g/\sqrt{\mathcal{E}_{\rm com}})} \text{ in mb } (10^{-28}\text{m}^2) \text{ with } B_g = 31.3970\sqrt{\text{keV}}$$

$$\mathcal{E}_{\rm com} = \mathcal{E}_{\rm rel}/2 = m_D v_{\rm rel}^2/4$$

Neutron rate yielded by a single fast ion marker:

$$R = n_D \sigma v_{\rm rel}$$

Emitted neutron parameters:

$$\mathbf{v_n} = \mathbf{v}_{\rm com} + \mathbf{u_n}$$

$$\langle \mathcal{E}_n \rangle \simeq 2.45\,\text{MeV} \text{ with } |\mathbf{u_n}| \simeq 2.165 \cdot 10^7\text{m/s}.$$

Isotropy of the neutron emission

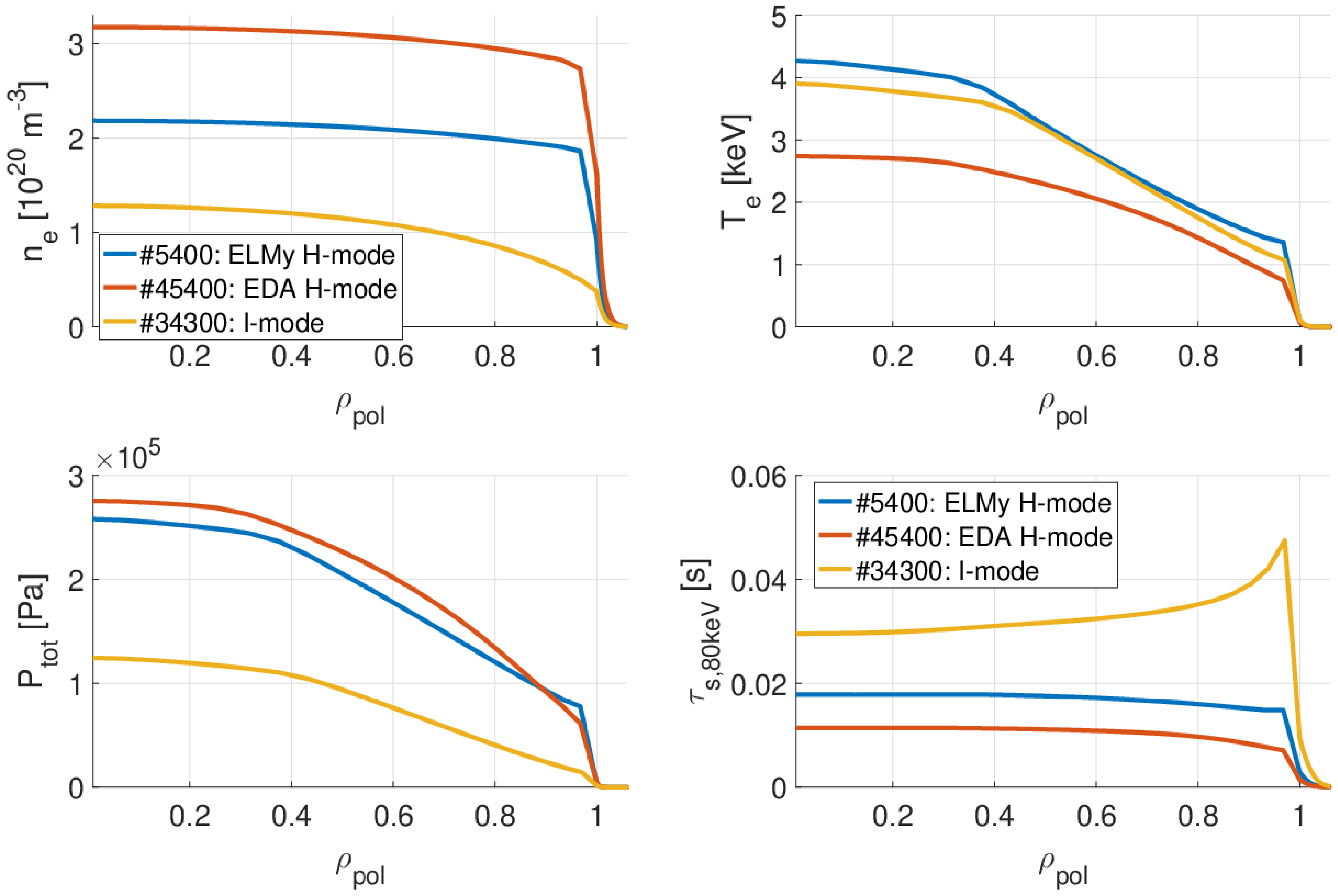
$$\begin{cases} u_{nX} = |\mathbf{u_n}| \cos(\arcsin(\chi)) \cos(\alpha) \\ u_{nY} = |\mathbf{u_n}| \cos(\arcsin(\chi)) \sin(\alpha) \\ u_{nZ} = |\mathbf{u_n}|(\chi) \end{cases}$$

STRATEGIES FOR 3 EDGE TRANSPORT BARRIERS TYPES

Thanks to scaling laws we recover the same type of profiles as was observed in ALCATOR C-mod

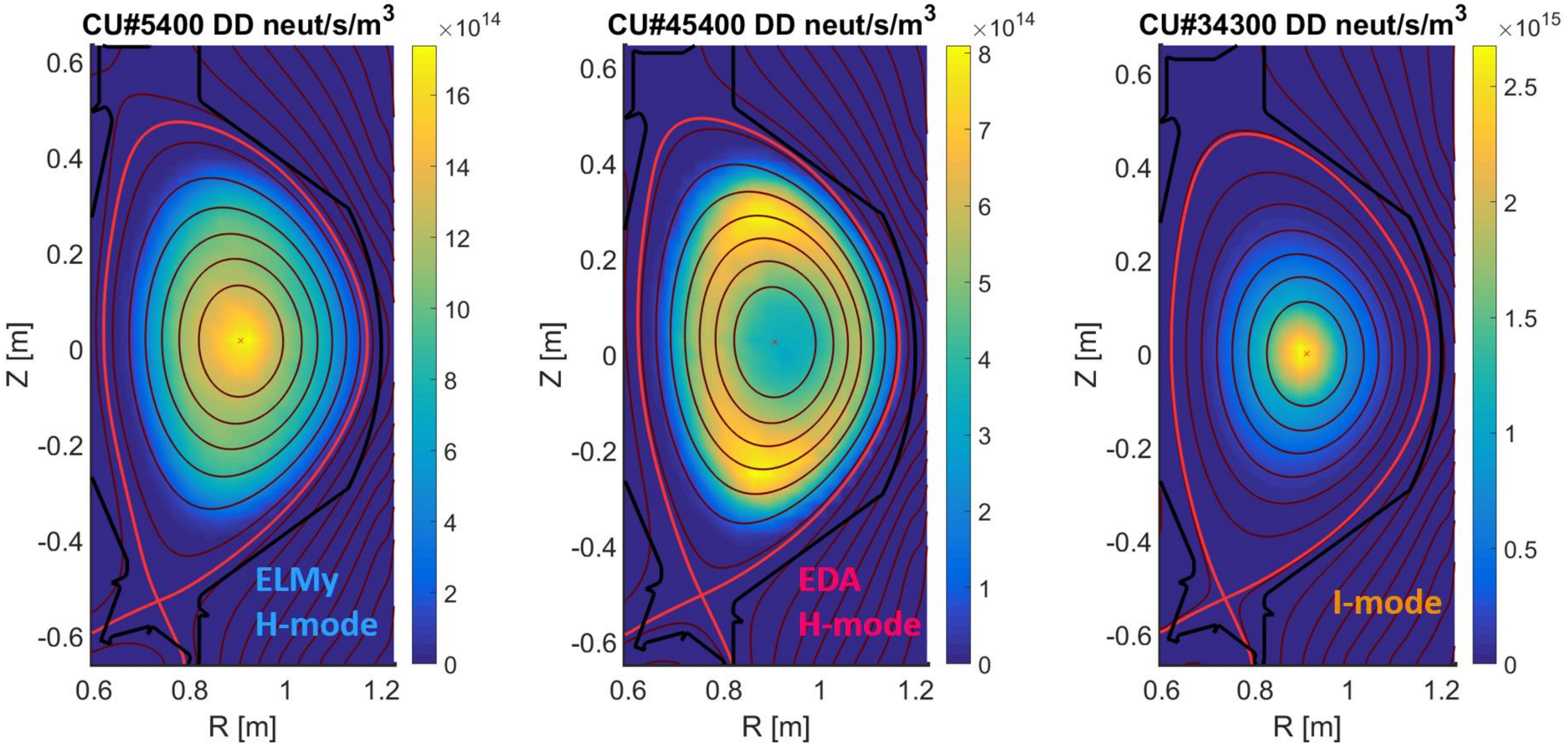
Scenario	β_N [%]	n_e [10^{20} m^{-3}]	τ_e [ms]	P_{ped} [kPa]	ν^*_{ped}	$n_{0,wall}$ [10^{18} m^{-3}]
#5400	1.24	2.04	83	77	0.3	3.5
#45400	1.3	3	135	62	1.2	8.6
#34300	0.9	1	39	15	0.3	0.7

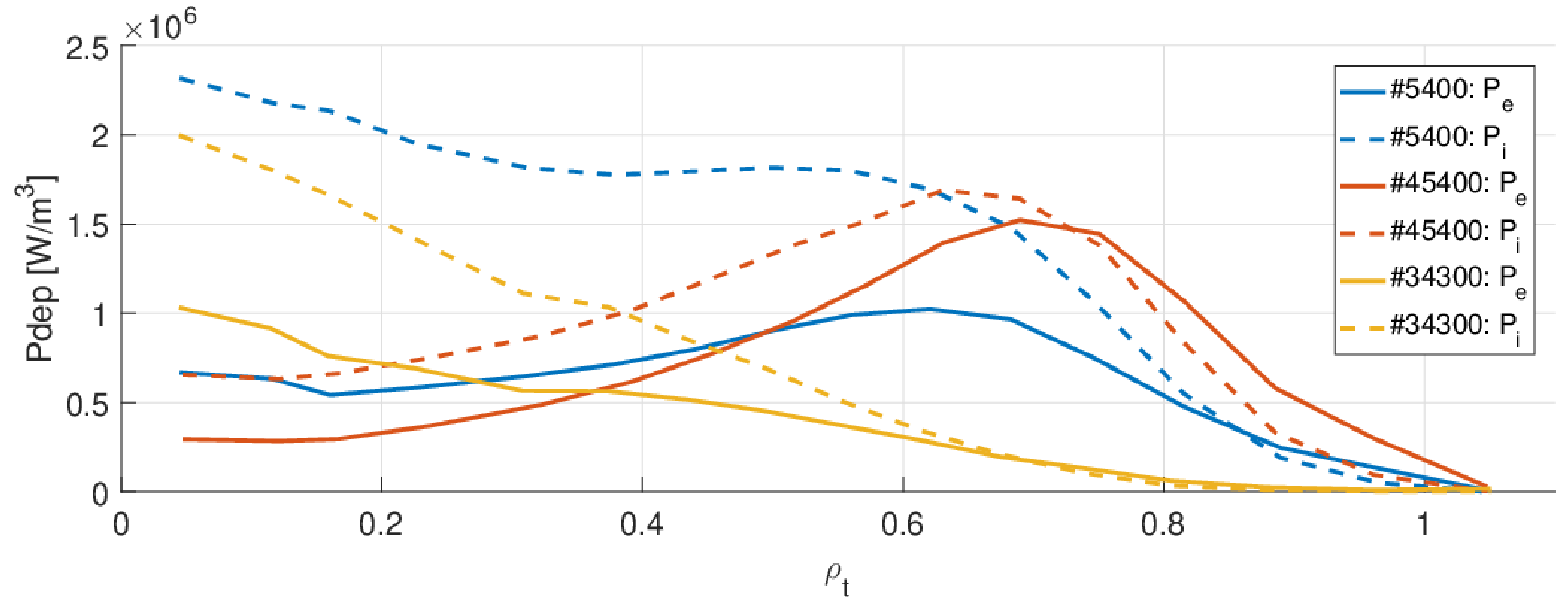
$$\tau_s = \frac{m_f}{m_e}\frac{1}{3\nu_e}\ln\left[1+\left(\frac{\mathcal{E}_0}{\mathcal{E}_{crit}}\right)^{3/2}\right] \text{ with } \mathcal{E}_{crit} \simeq 18.6T_e$$

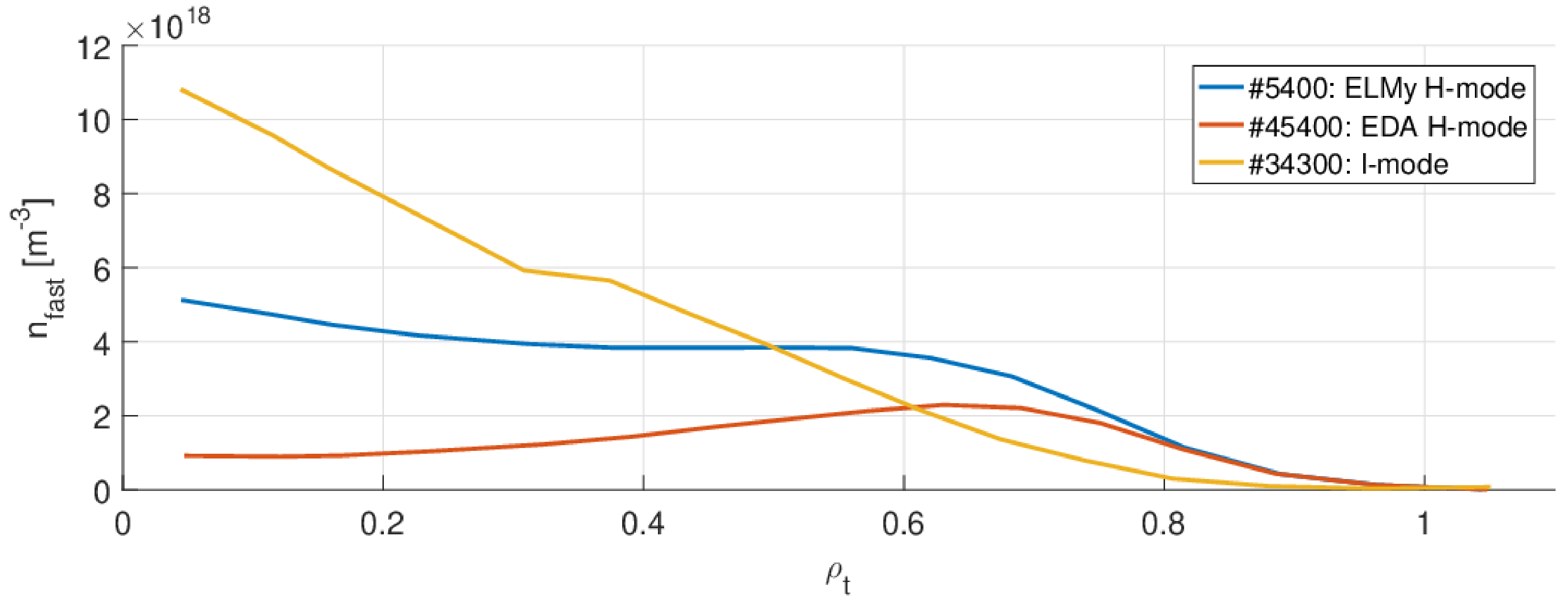


Full-orbit simulations with the EBdyna code allow to obtain description of neutrons in space and velocity space

EBdyna for 3-4MW NBI @R_t=0.6m

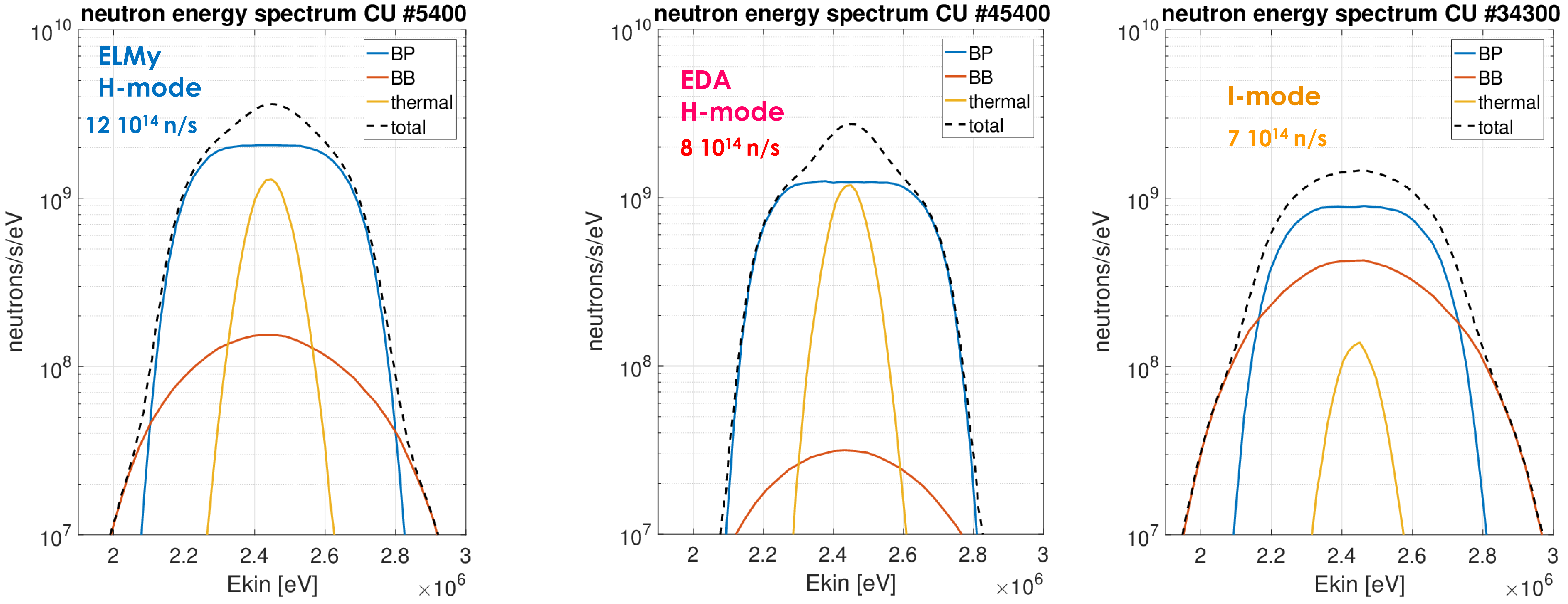






Scenario	EBdyna neutrons Thermal [10^{14} s^{-1}]	EBdyna neutrons Beam-Plasma [10^{14} s^{-1}]	EBdyna neutrons Beam-Beam [10^{14} s^{-1}]	EBdyna neutrons Total [10^{14} s^{-1}]	METIS neutrons Total [10^{14} s^{-1}]
#5400	2.2	9.9	0.8	12.9	13.7
#45400	1.8	6.1	0.2	8.1	9.0
#34300	0.2	4.1	2.3	6.6	7.9

Full-orbit simulations with the EBdyna code allow to obtain description of NBI in space and velocity space



Full-orbit simulations with Ebdyna (ex: CU#5400): generates 4.2M steady state neutron markers (BB+BP)

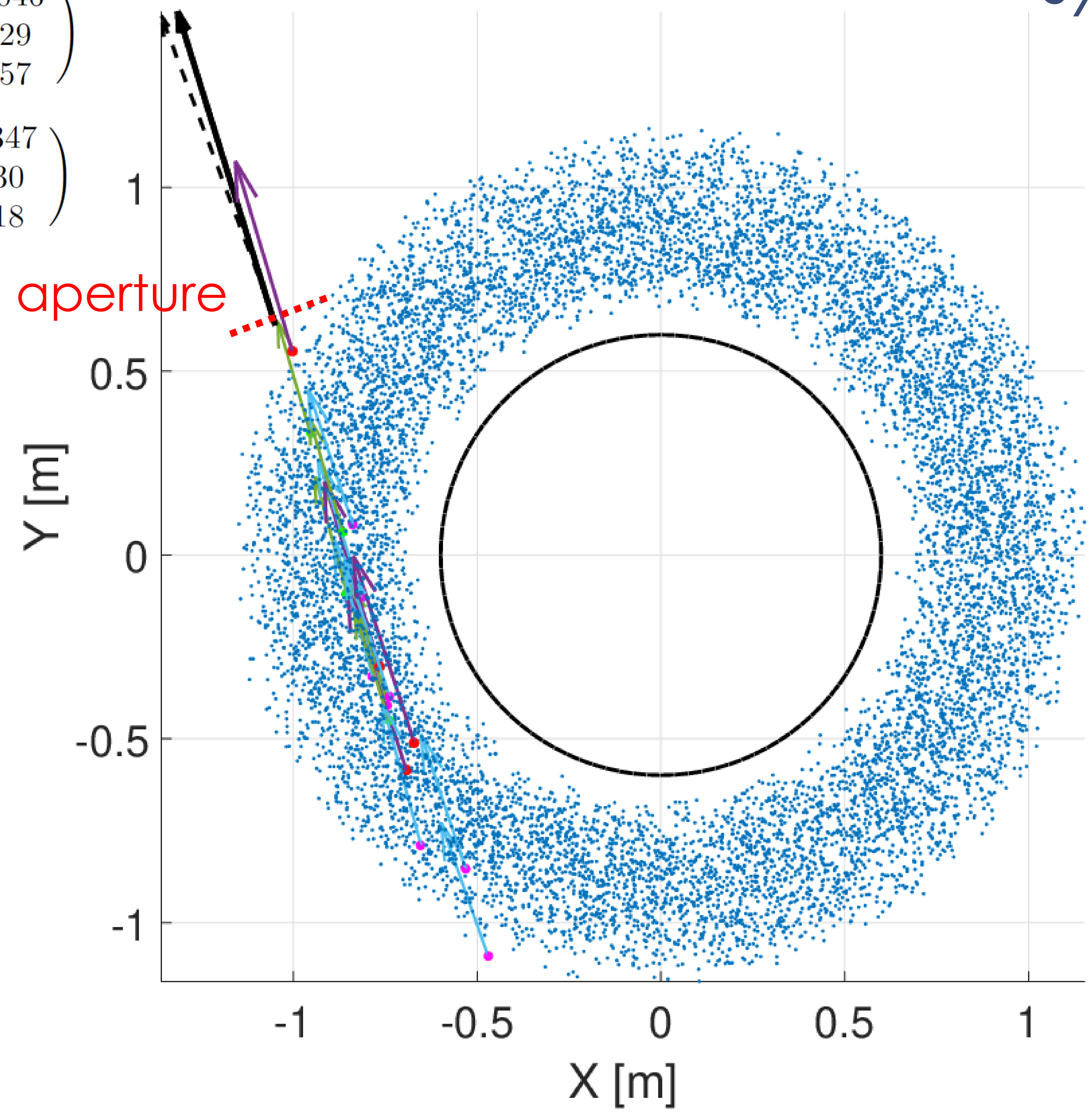
Detailed map of the neutron source is obtained from simulation : can design synthetic diagnostics for neutron camera

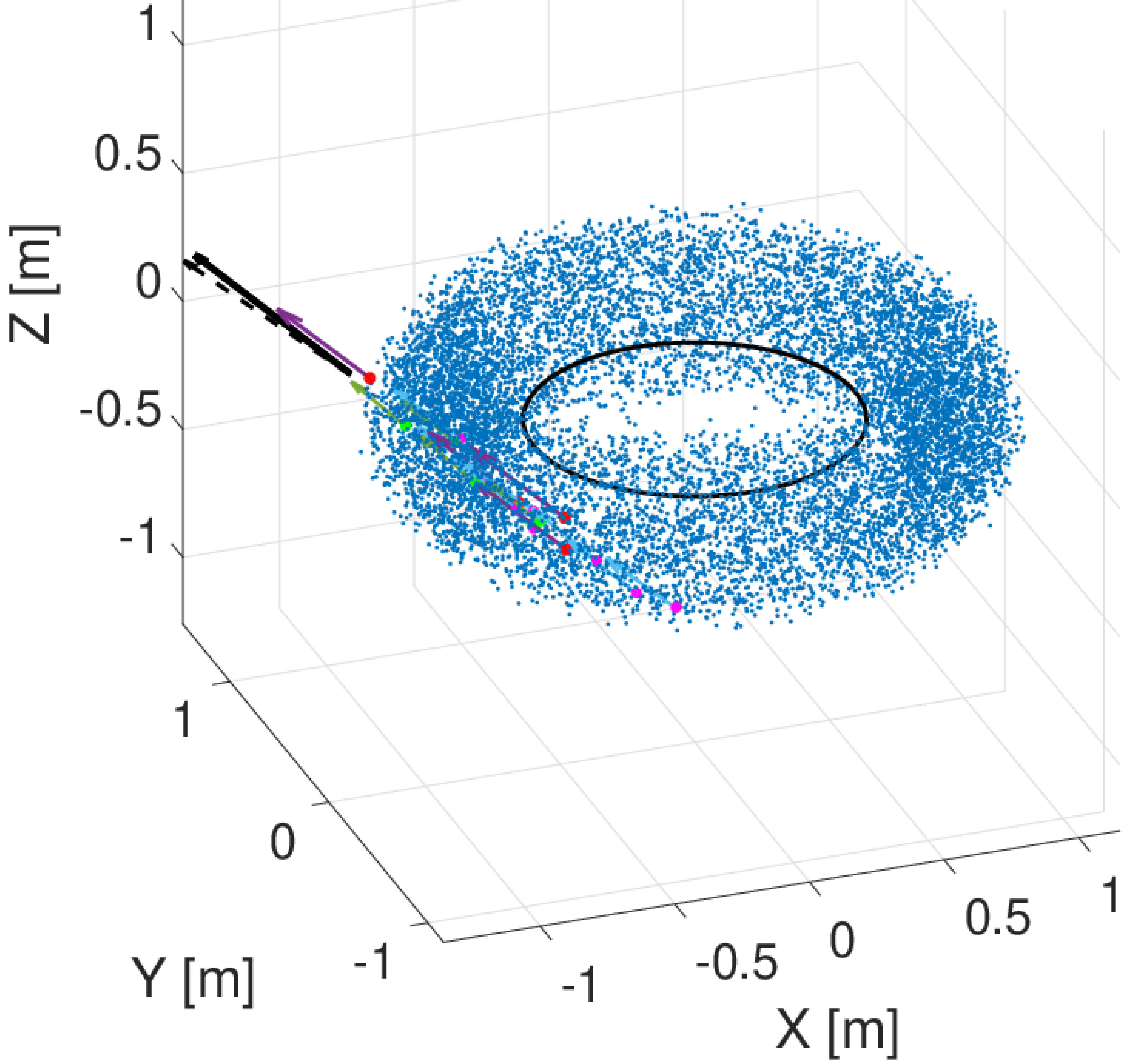
$$X_{\mathrm{LOS}\#1} = \begin{pmatrix} -1.066 \\ 0.682 \\ 0.064 \end{pmatrix} \text{ and } X_{\mathrm{LOS}\#2} = \begin{pmatrix} -1.046 \\ 0.629 \\ 0.057 \end{pmatrix}$$

$$\mathbf{v}_{\mathrm{LOS}\#1} = \begin{pmatrix} -0.295 \\ 0.948 \\ 0.12 \end{pmatrix} \text{ and } \mathbf{v}_{\mathrm{LOS}\#2} = \begin{pmatrix} -0.347 \\ 0.930 \\ 0.118 \end{pmatrix}$$

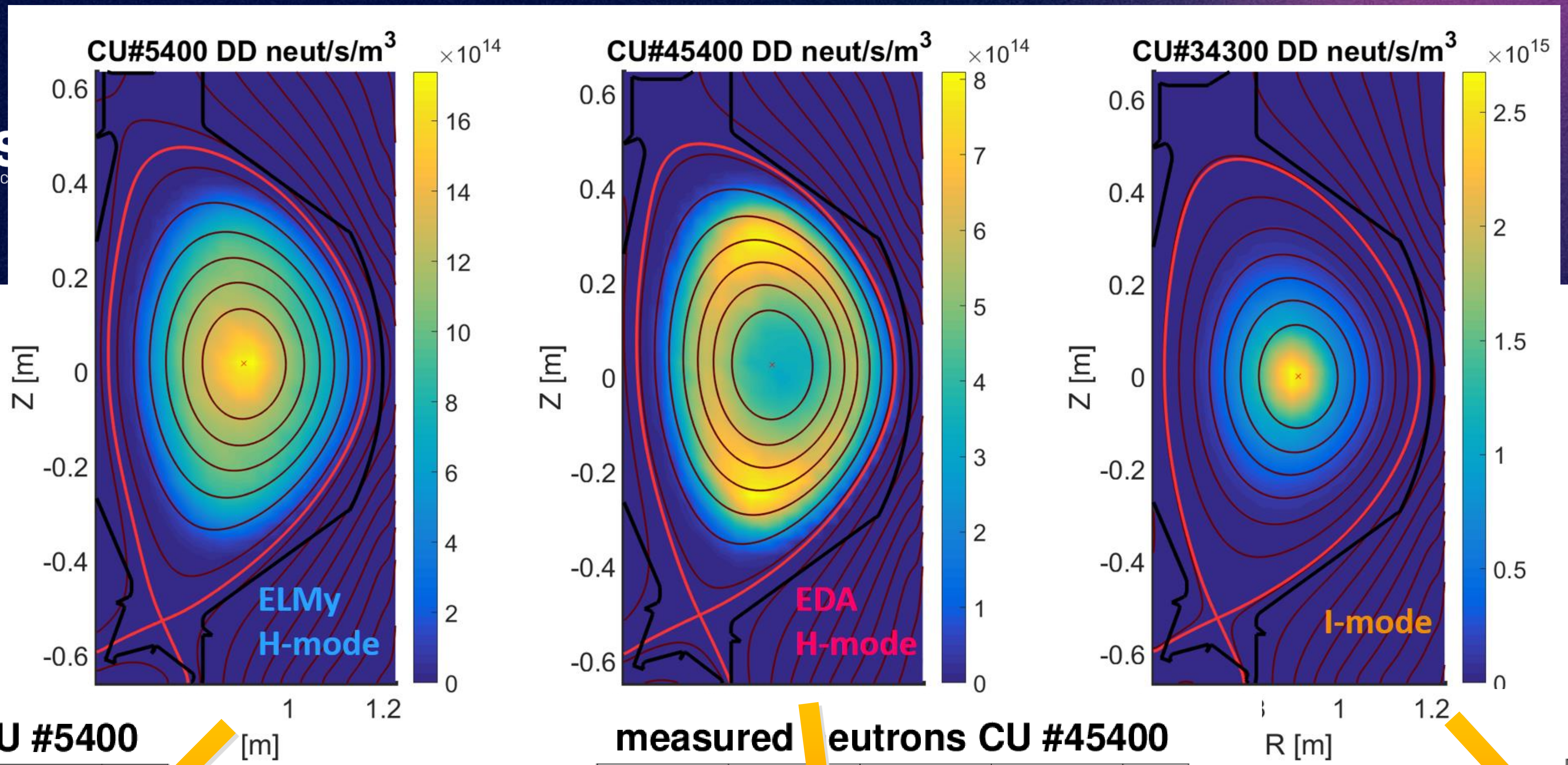
Aperture radius ~0.06m

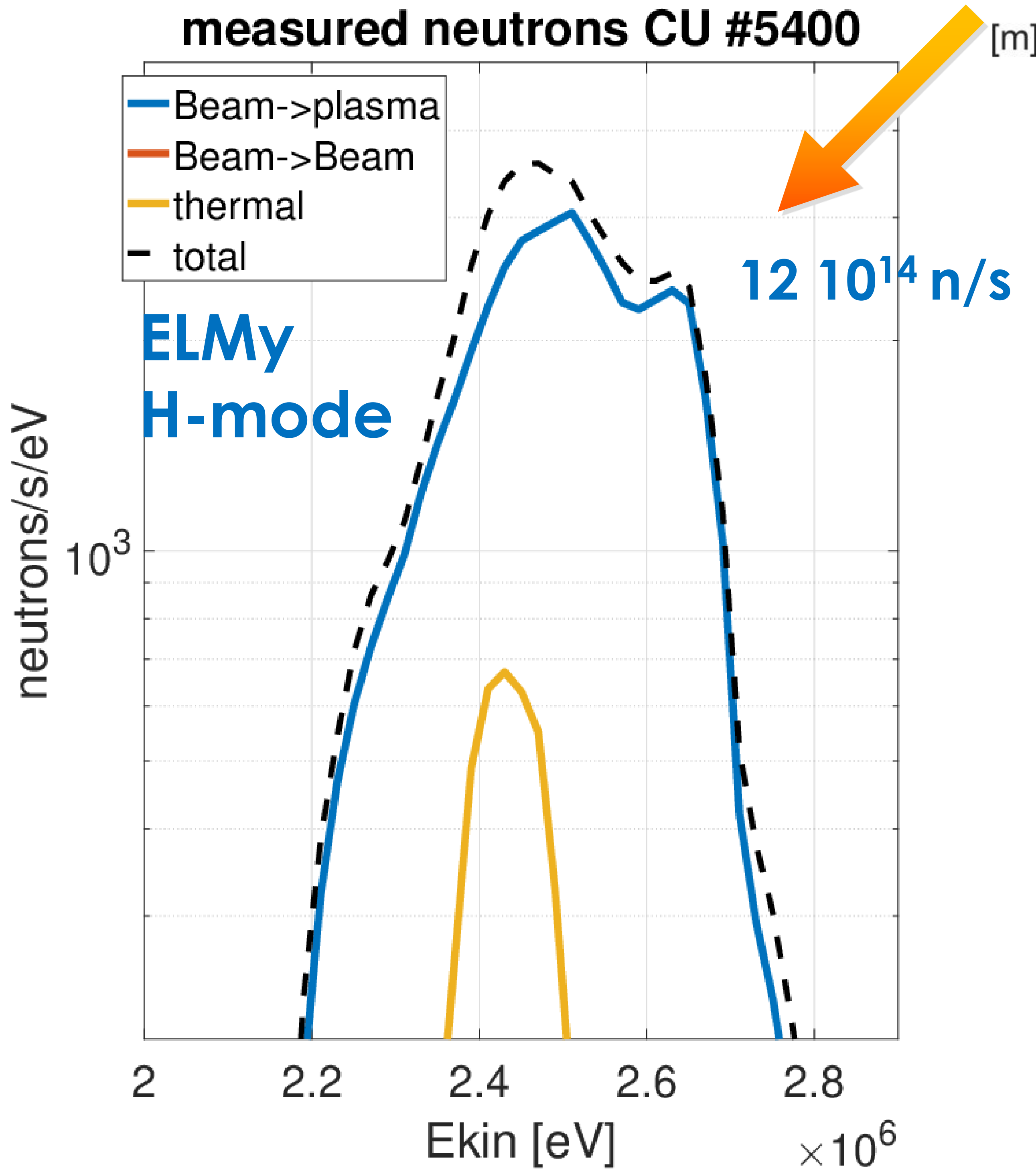
Collimation length ~1.6m
-> cos($\mathbf{v}_n$,$\mathbf{v}_{Detect}$)>0.9993

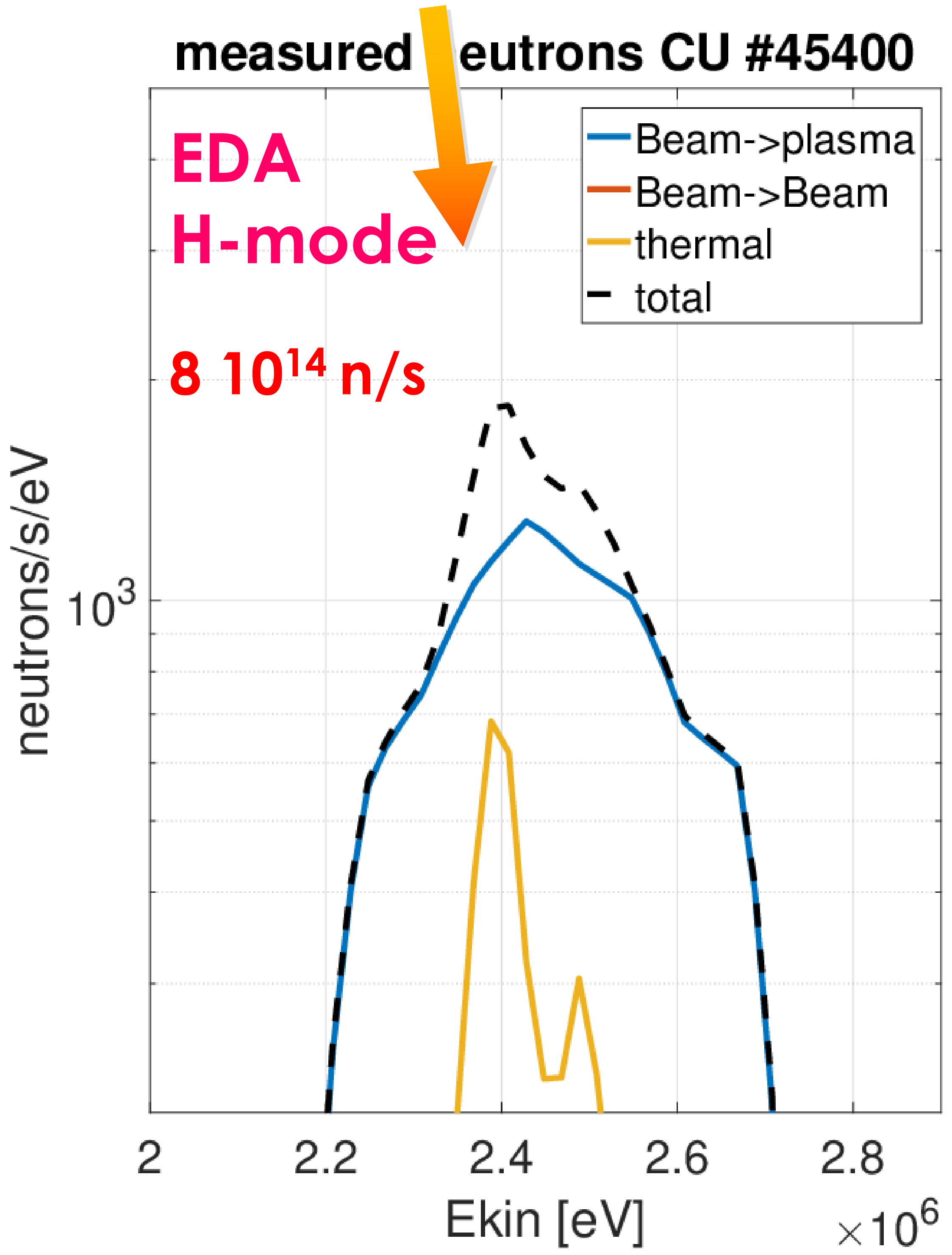


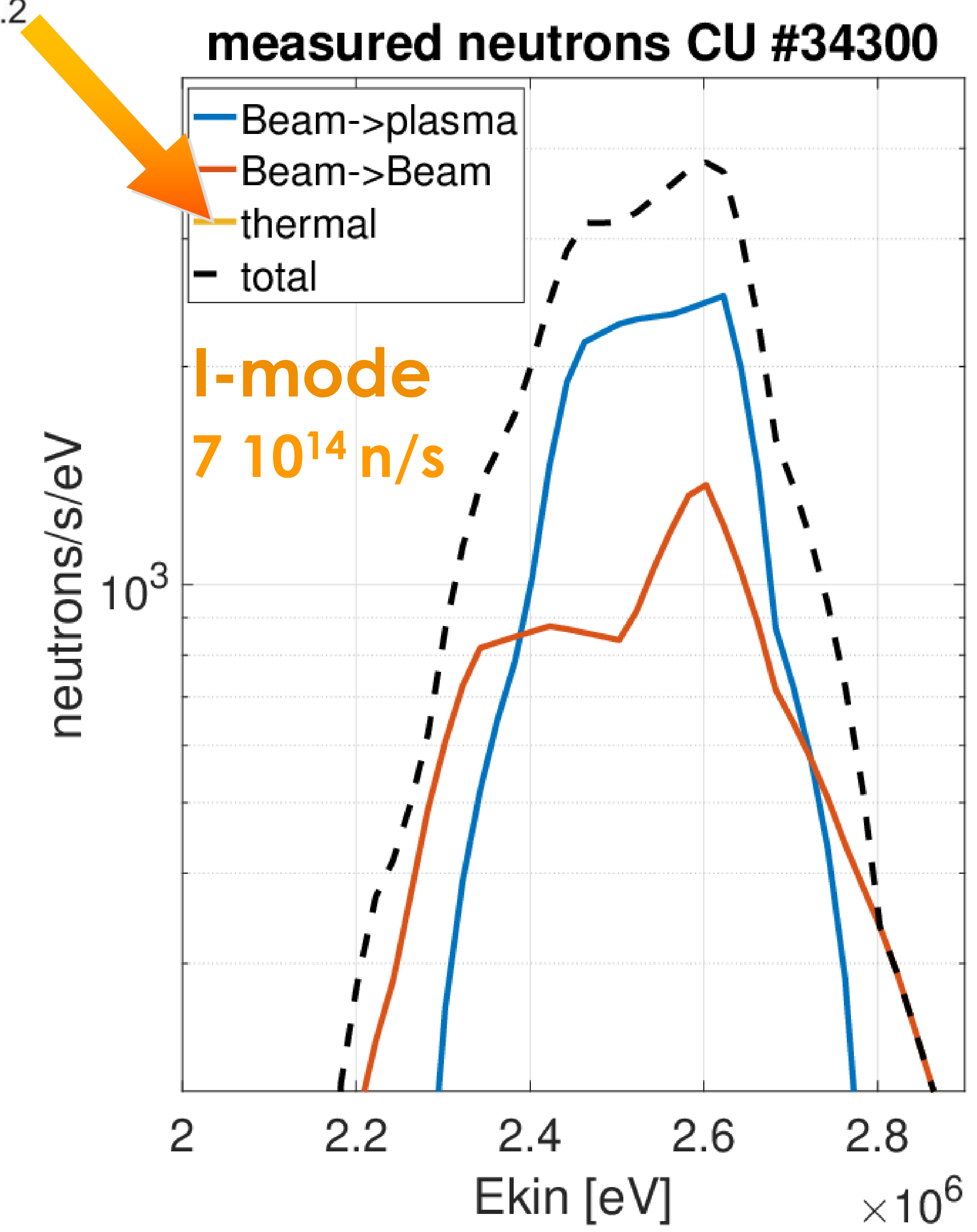


NDD NEUTRONS









Overview of neutron generation with EBdyna

- **Full kinetic neutron calculations implemented in EBdyna**
- **Neutron fluxes & spectra in EBdyna in good agreement with theoretical formula and METIS calculations**
- **Large amount of neutron markers allow to derive approximate statistics for neutron flux on diagnostics and measured neutron spectra: importance of absolute calibration to avoid bias due to poloidal distribution of neutron source**

Summary & outlook

- **NBI is a large device used to heat up both electrons and ions in tokamaks. Its main parameters are the tangency radius and the injection energy.**

- **Modelling particle orbits in tokamaks: the Boris algorithm can be implemented in a fast collisionless orbit solver in toroidal geometry. Typical required time step are of order 10^{-9} s for NBI ions.**

- **Coulomb Collisions simplified operators can be added on top of the collisionless solver in order to give a representation of slowing down and pitch angle scattering. Obtaining steady state requires simulations of order 10-1500 ms: can become challenging computationally.**

- **The full distribution of fast ions has many application: modelling of fast neutrals generation in COMPASS Upgrade (fast ions diagnostics). It can also be used for MHD studies.**

- *Further, modelling of NBI duct losses [see extra slides!]*

F. Jaulmes et al 2022 PPCF 64 125001

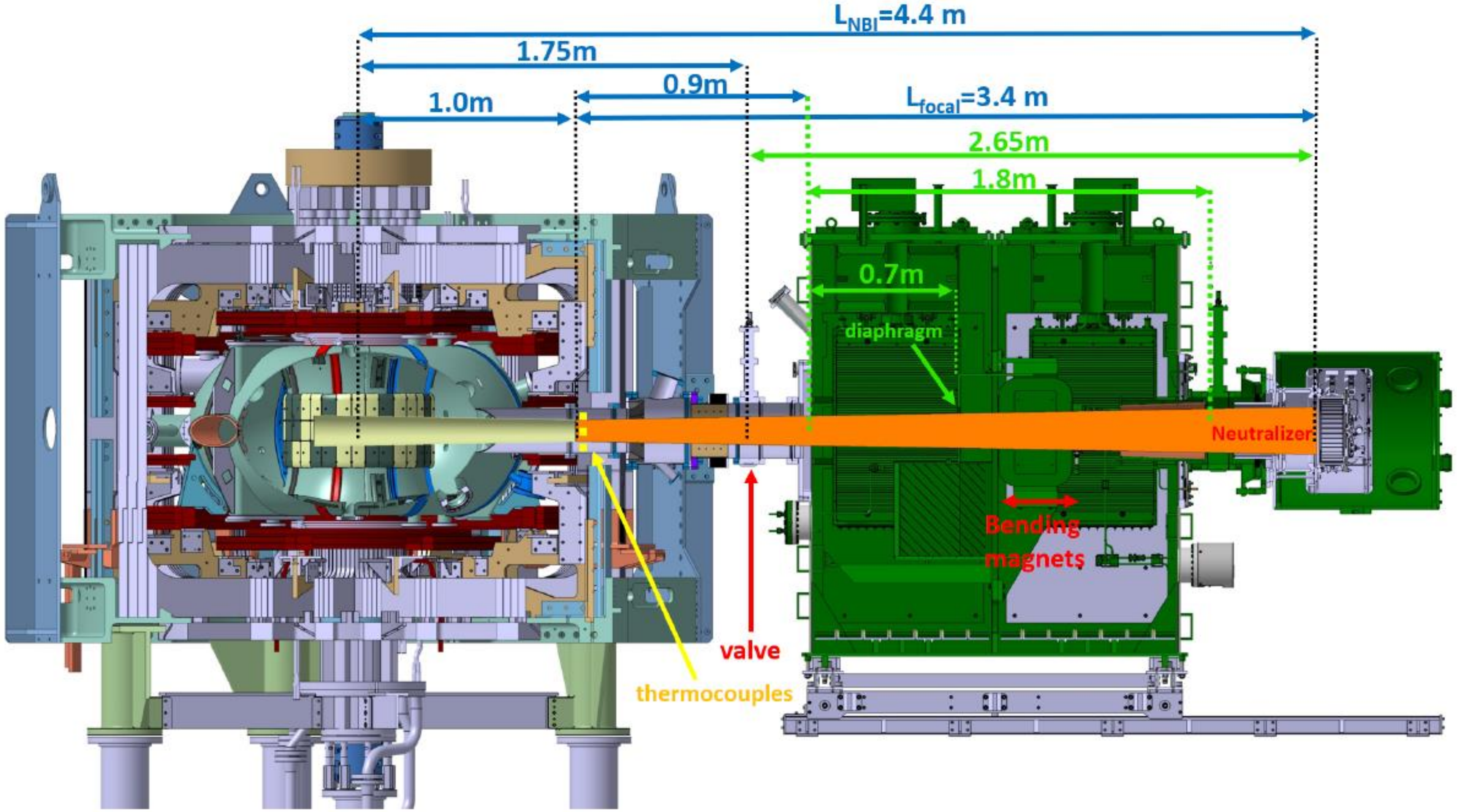
INSTITUTE OF PLASMA PHYSICS
OF THE CZECH ACADEMY OF SCIENCES

THANK YOU FOR YOUR ATTENTION

FABIEN JAULMES

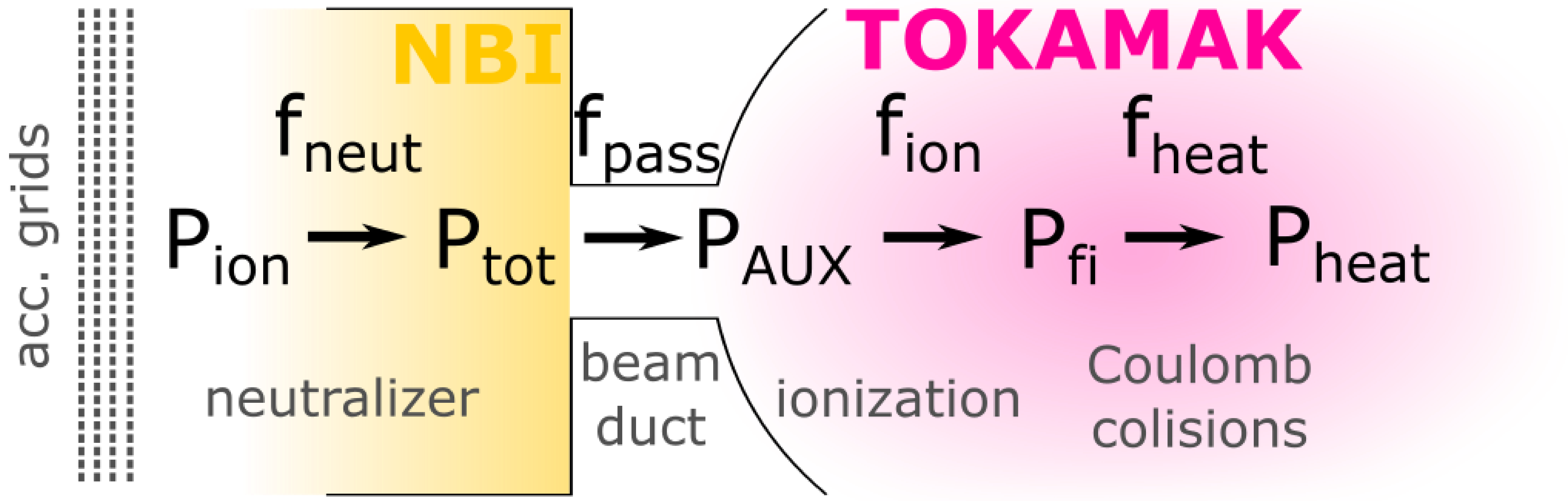
MINISTRY OF EDUCATION, YOUTH AND SPORTS

Overview of the dimensions of the NBI 0 in COMPASS [side view]



F. Jaulmes et al 2022 PPCF 64 125001

NBI POWERS



f_{neut}… neutralization efficiency
f_{pass} … passing fraction
f_{ion} … ionization efficiency
f_{heat} … heating efficiency

$P_{ion} = U_{beam} I_{beam}$ power of accelerated ions
P_{tot} …… total power produced in the neutrals
P_{AUX} …. Auxiliary heating power entering the tokamak
P_{fi} ….. Power in the formed fast ions
P_{heat}….. power heating the bulk plasma

Pitch angle scattering

Diffusion in the perpendicular plane (often called σ_{90} or σ) is different than momentum!

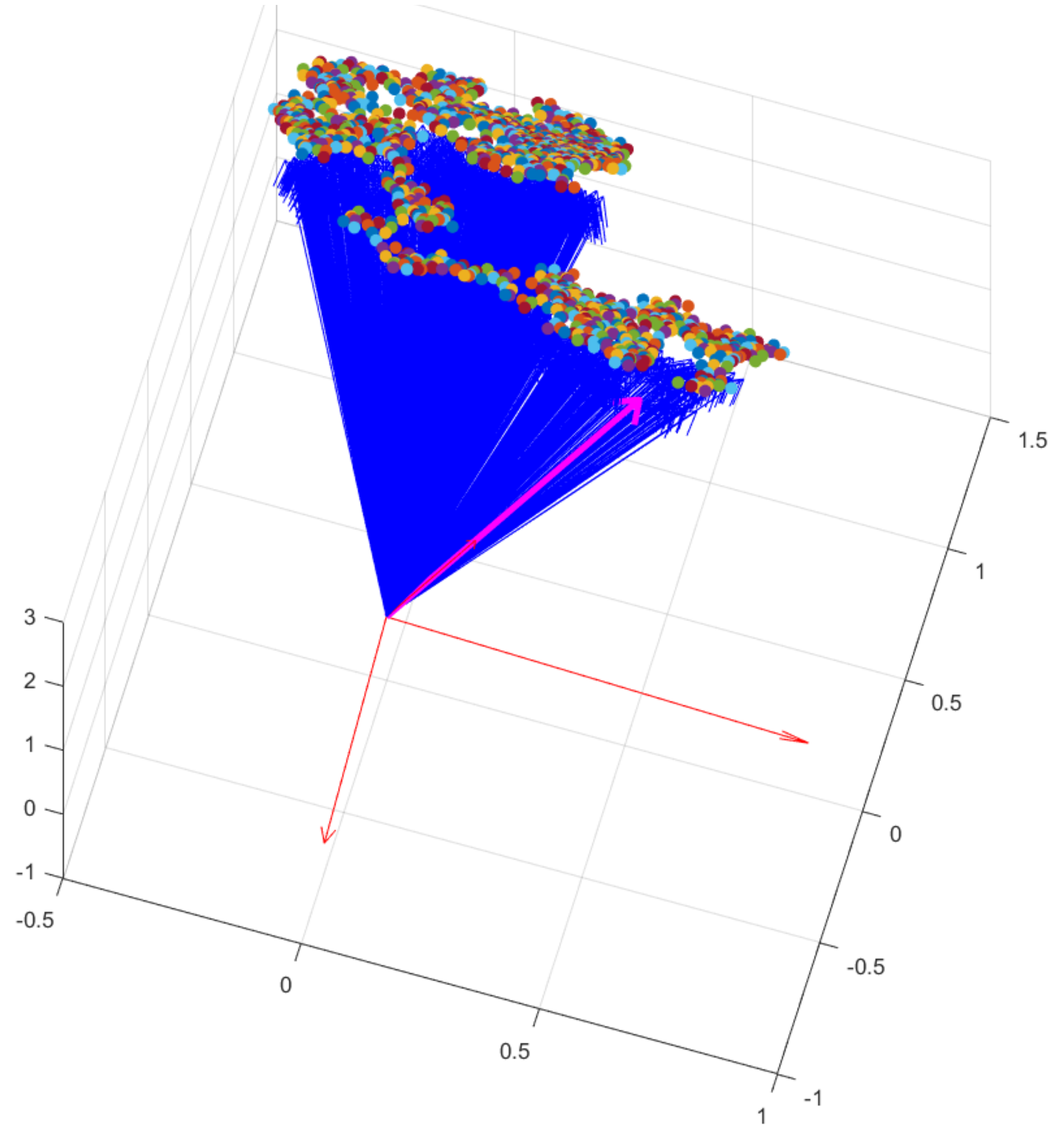
$$\sigma_{12} = \frac{2m_2}{m_1 + m_2}\nu_{12}$$

Momentum loss rate

$$\sigma_{be} = \frac{2m_e}{m_D + m_e}\nu_{be} \simeq 0 \qquad \sigma_{bi} = \frac{2m_D}{m_D + m_D}\nu_{bi} = \nu_{bi}$$

See: https://ocw.mit.edu/courses/nuclear-engineering/22-611j-introduction-to-plasma-physics-i-fall-2006/readings/chap3.pdf

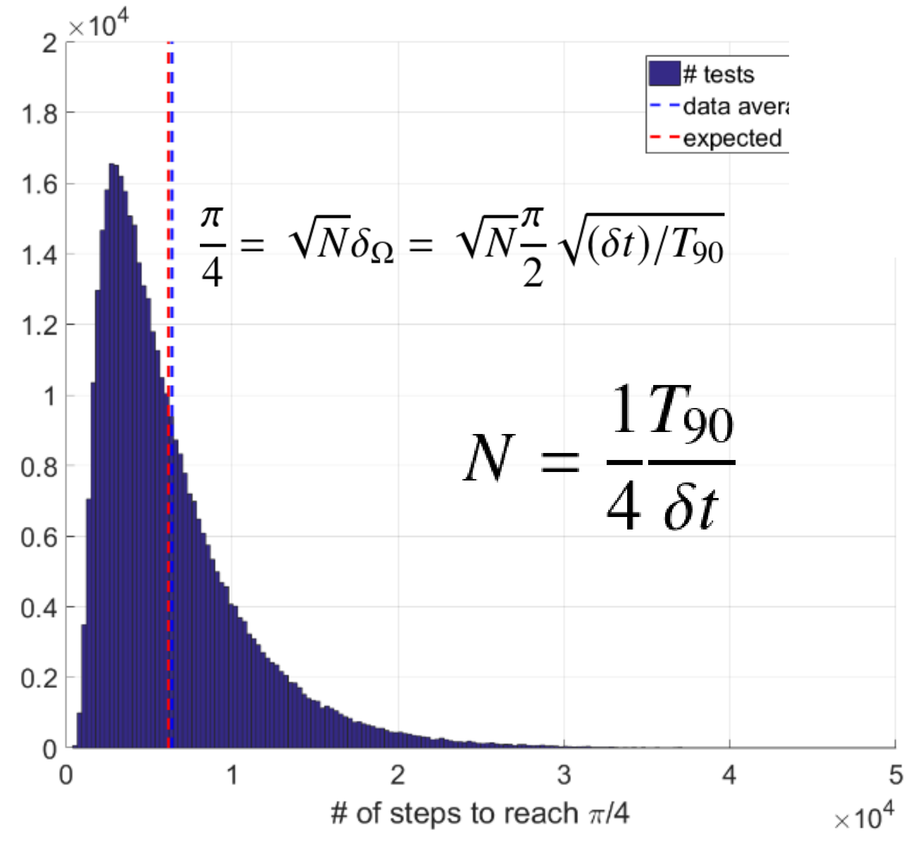
$$\delta\Omega = \mathcal{N}(1,1)\frac{\pi}{2}\sqrt{(\delta t)\nu_{\perp i}}$$



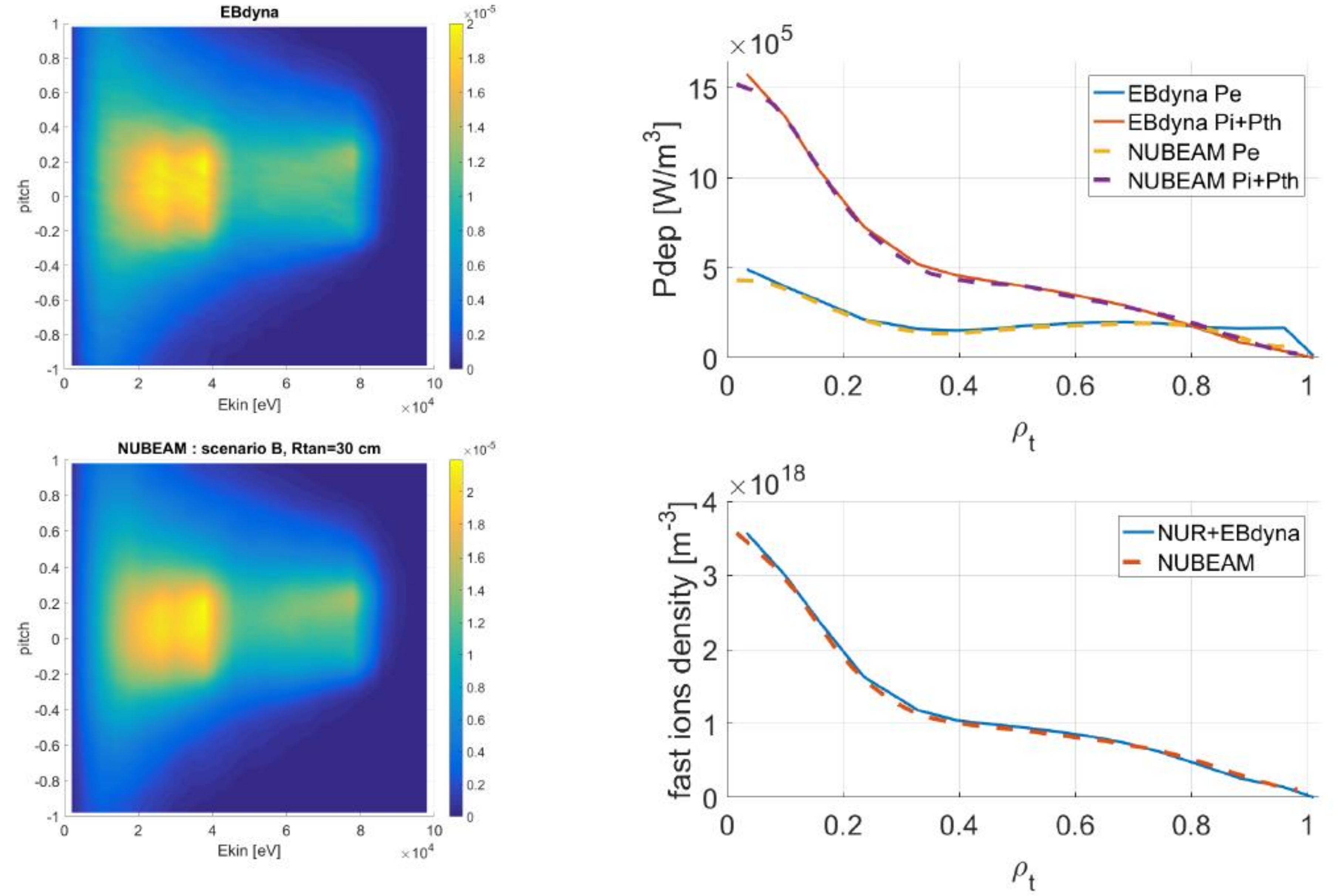
Simple scatter using systematic deviation:

$$\delta\Omega = \frac{\pi}{2}\sqrt{(\delta t)\sigma_{bi}}$$

Random walk in 3D



Benchmark with NuBeam code:

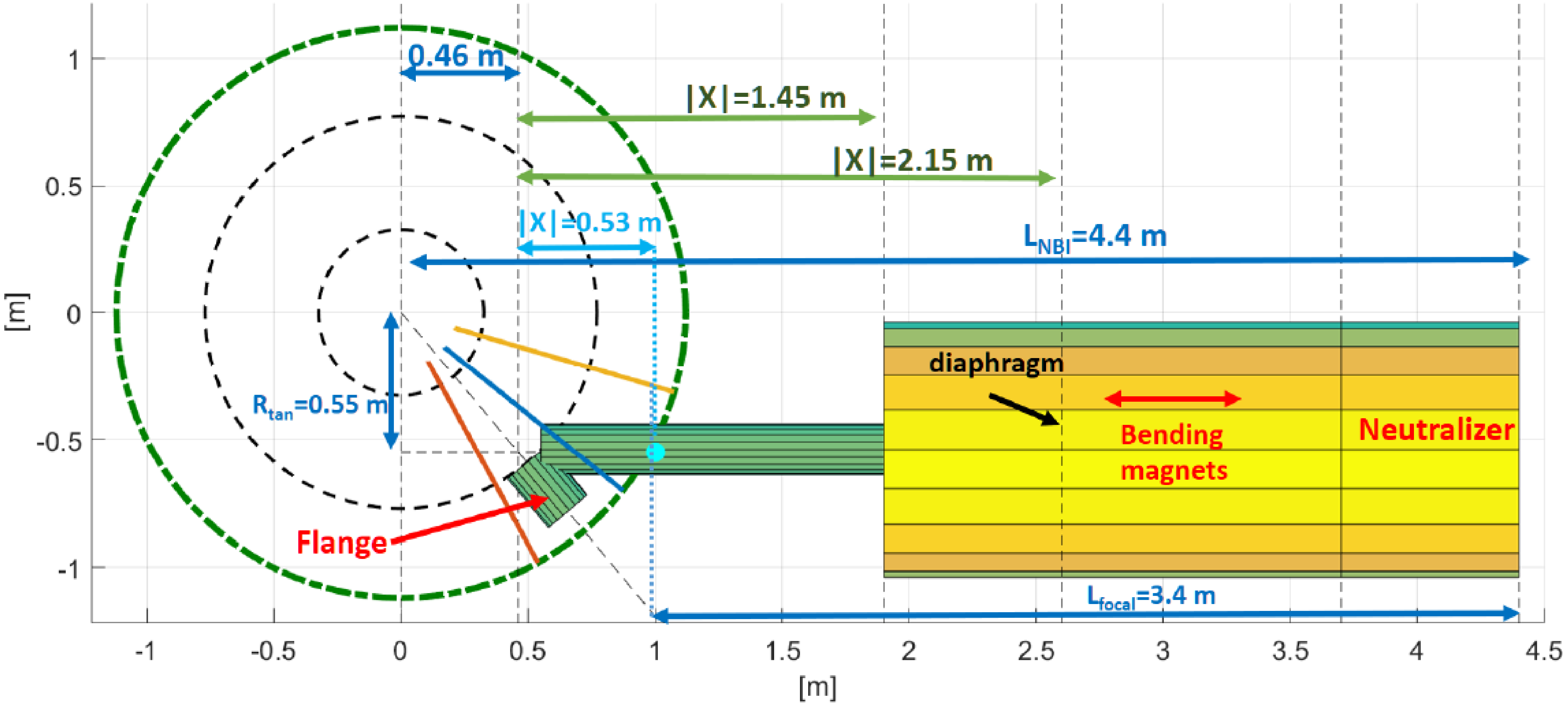


F. Jaulmes et al 2021 Nucl. Fusion 61 046012

Beam duct modelling

- Modelling of fast neutrals generation in COMPASS
  - Ray-tracing of fast neutrals generation
  - Calculation of position of ionized neutral markers
  - Full orbit calculation of ion trajectory in 3D field

Overview of the dimensions of the NBI 0 in COMPASS\ [top view]

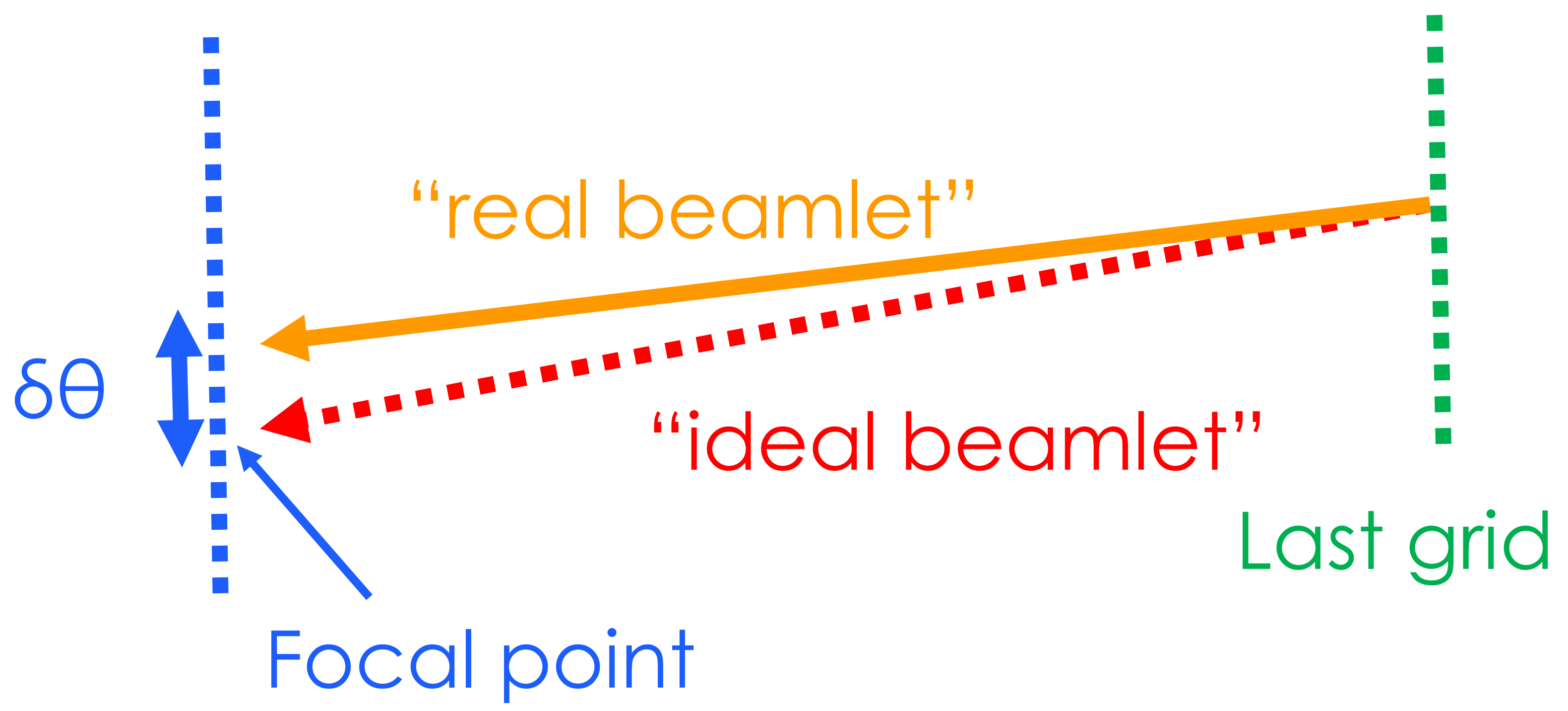


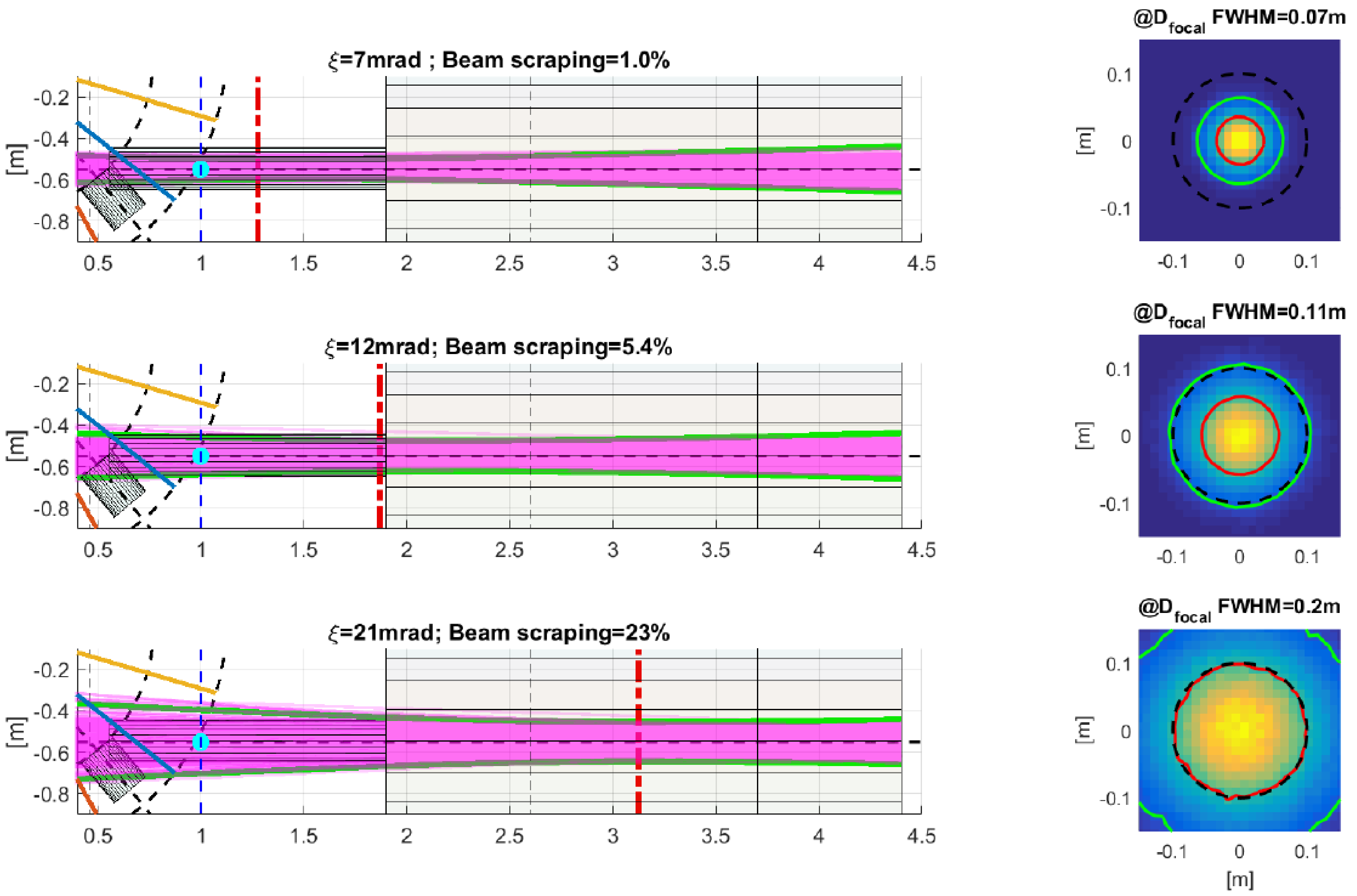
F. Jaulmes et al 2022 PPCF 64 125001

Ray-tracing of fast neutrals

$$f(t_g) = \frac{1}{\xi\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{t_g}{\xi}\right)^2\right)$$

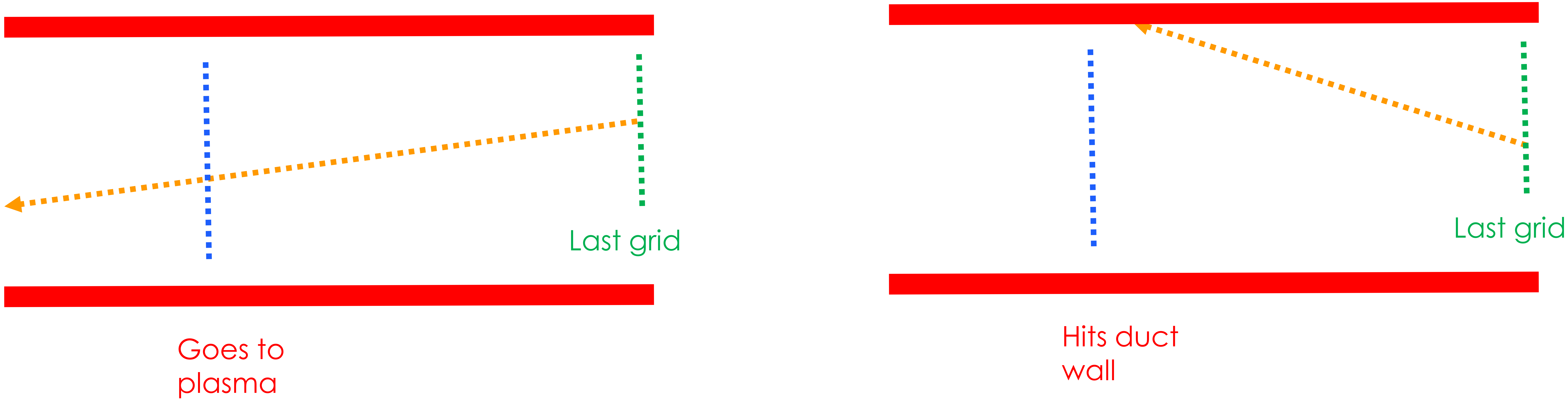
$$t_g = \tan(\delta\theta) \simeq \delta\theta$$



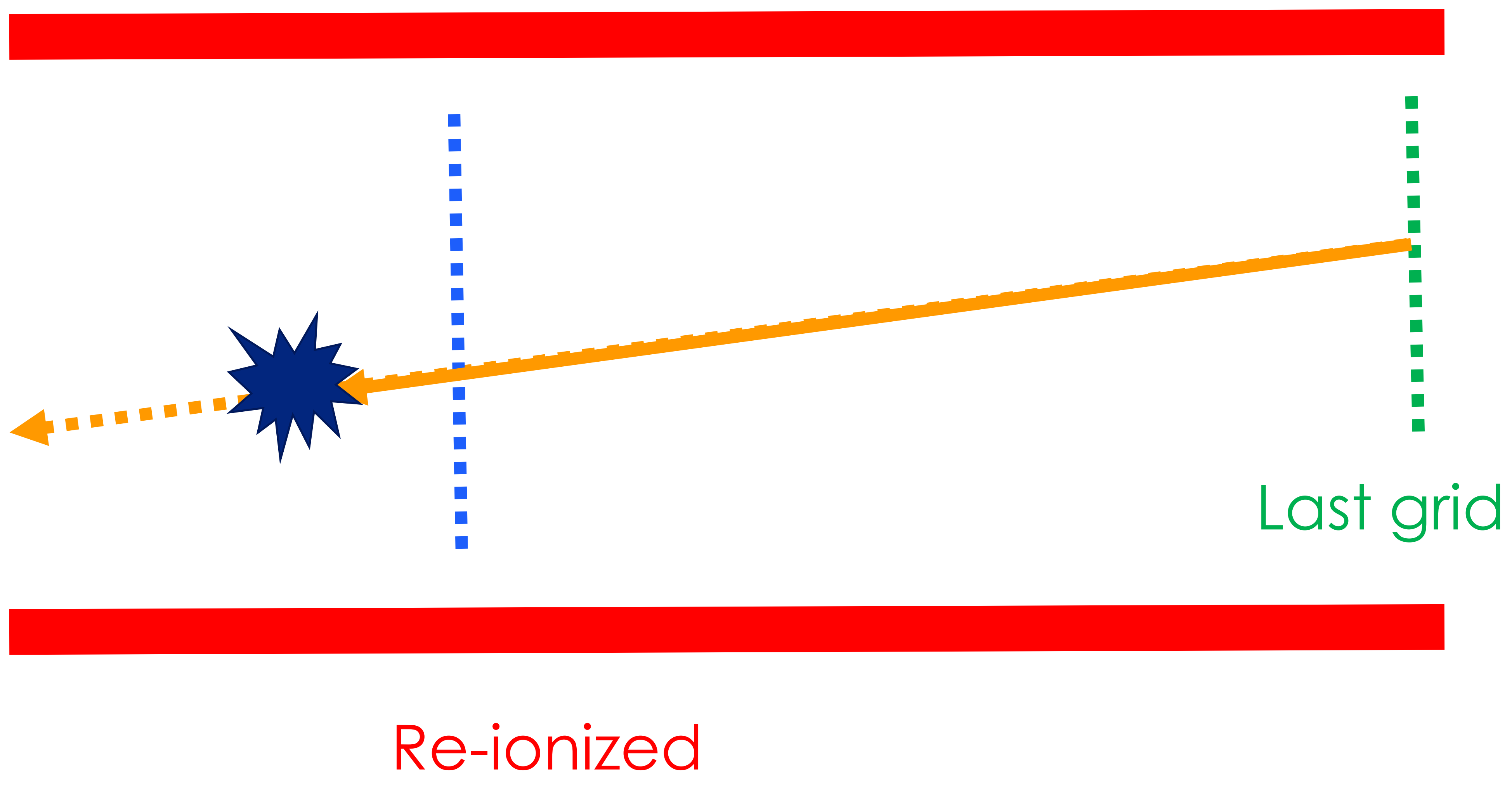


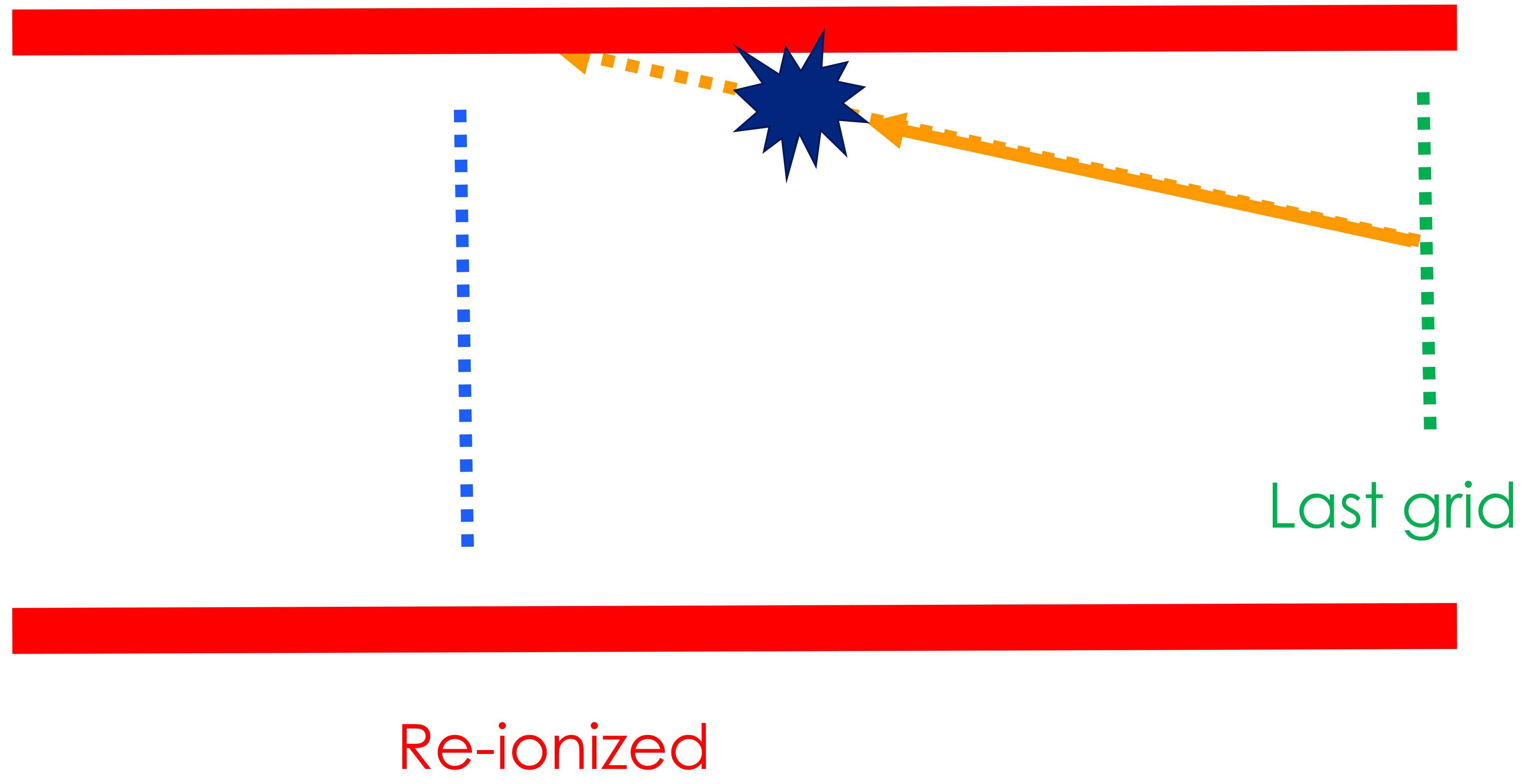
F. Jaulmes et al 2022 PPCF 64 125001

Fast neutrals & duct walls



Ionizations of fast neutrals

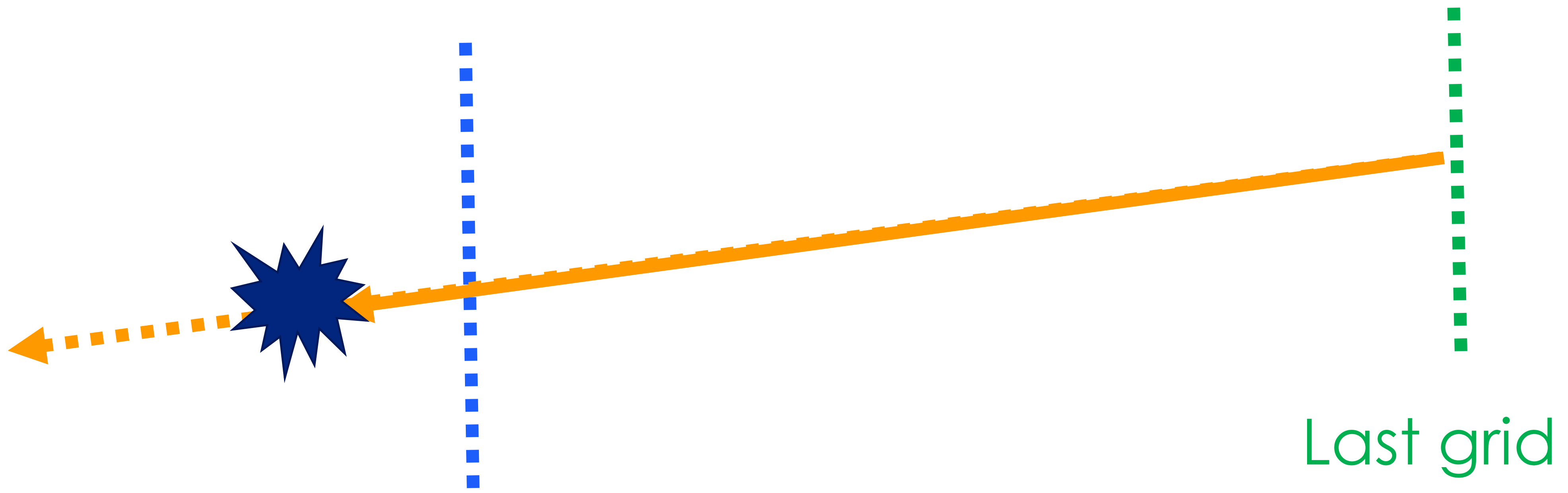


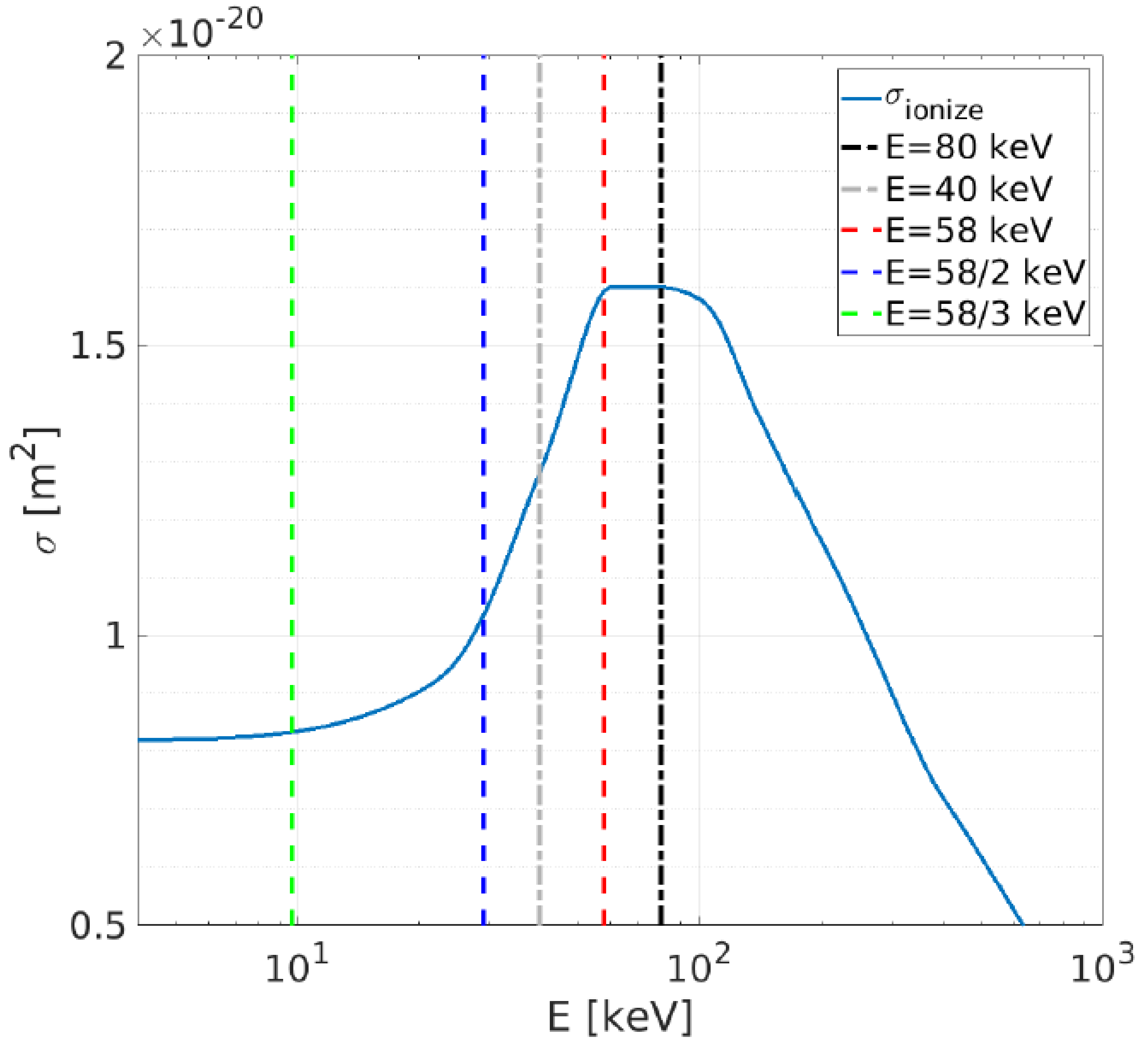


But the duct contains residual neutral gas! ~ 10^{18} m-$^{-3}$

Ionizations of fast neutrals

$$\frac{\mathrm{d}\mathcal{P}_n}{\mathrm{d}l} = -\mathcal{P}_n n_0 \sigma(\mathcal{E})$$

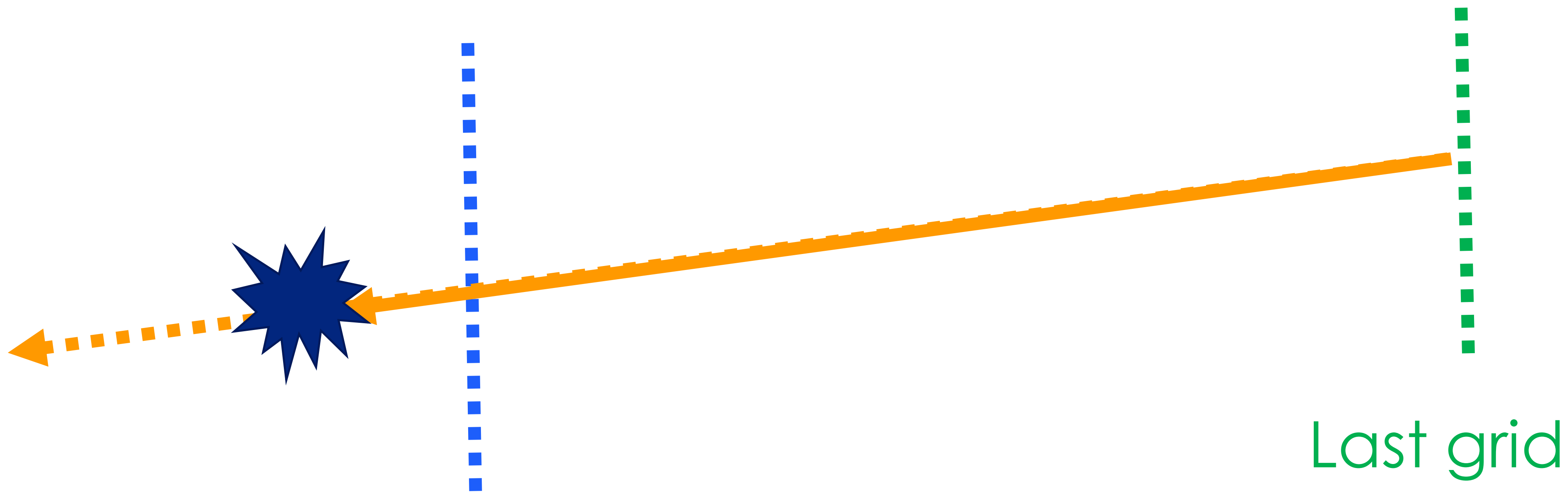




Ionizations of fast neutrals

$$f(t_g) = \frac{1}{\xi\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{t_g}{\xi}\right)^2\right)$$

$$t_g = \tan(\delta\theta) \simeq \delta\theta$$



$$\frac{\mathrm{d}\mathcal{P}_n}{\mathrm{d}l} = -\mathcal{P}_n n_0 \sigma(\mathcal{E})$$

$$\mathcal{P}_n = \exp\left(-\int n_0\sigma(\mathcal{E})\mathrm{d}l\right)$$

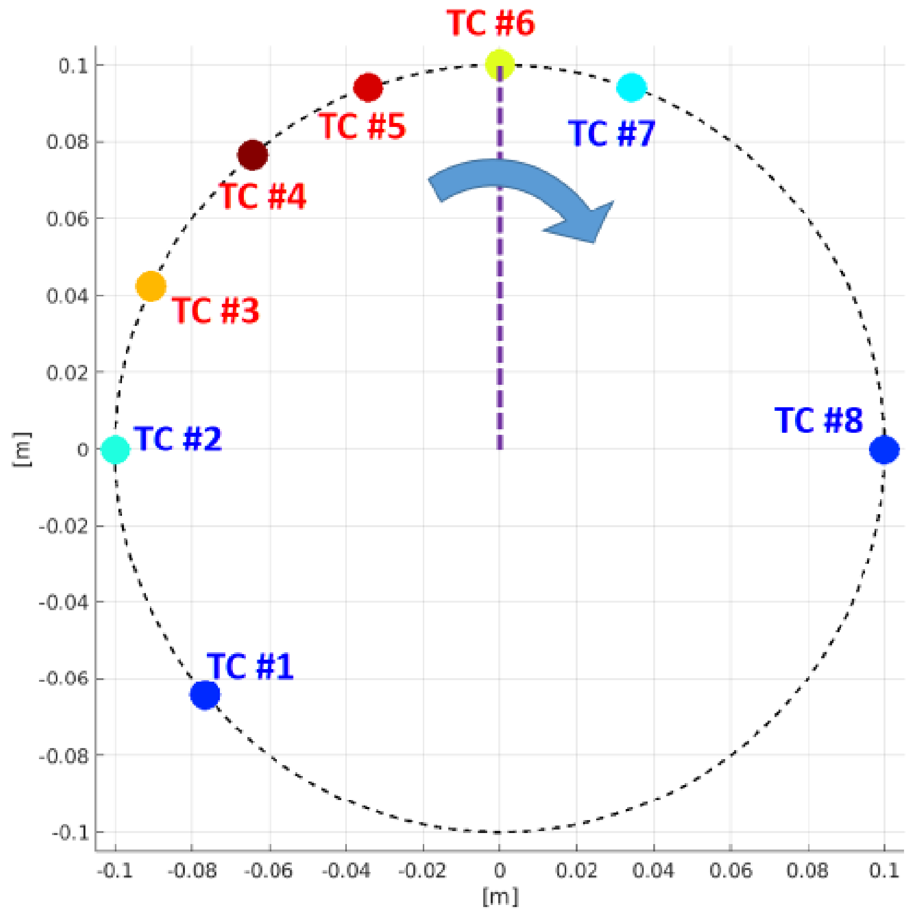
$$\mathcal{P}_i(l) = 1 - \mathcal{P}_n(l)$$

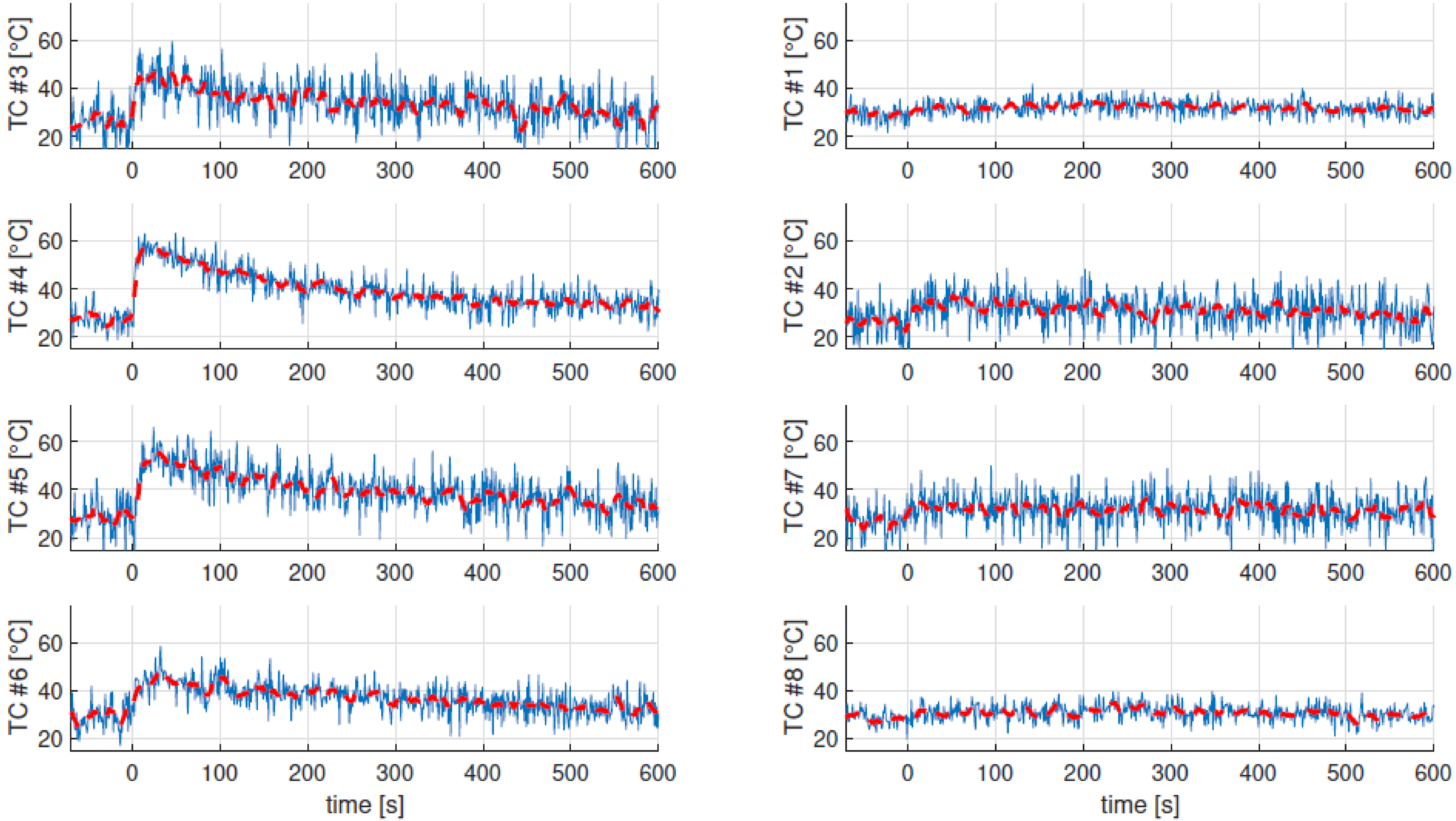
$$\mathcal{P}_i - \mathcal{R} = 0;$$

$$I = I_0\left(\exp\left(-L n_0 \sigma_i\right)\right)$$

(uniform density)

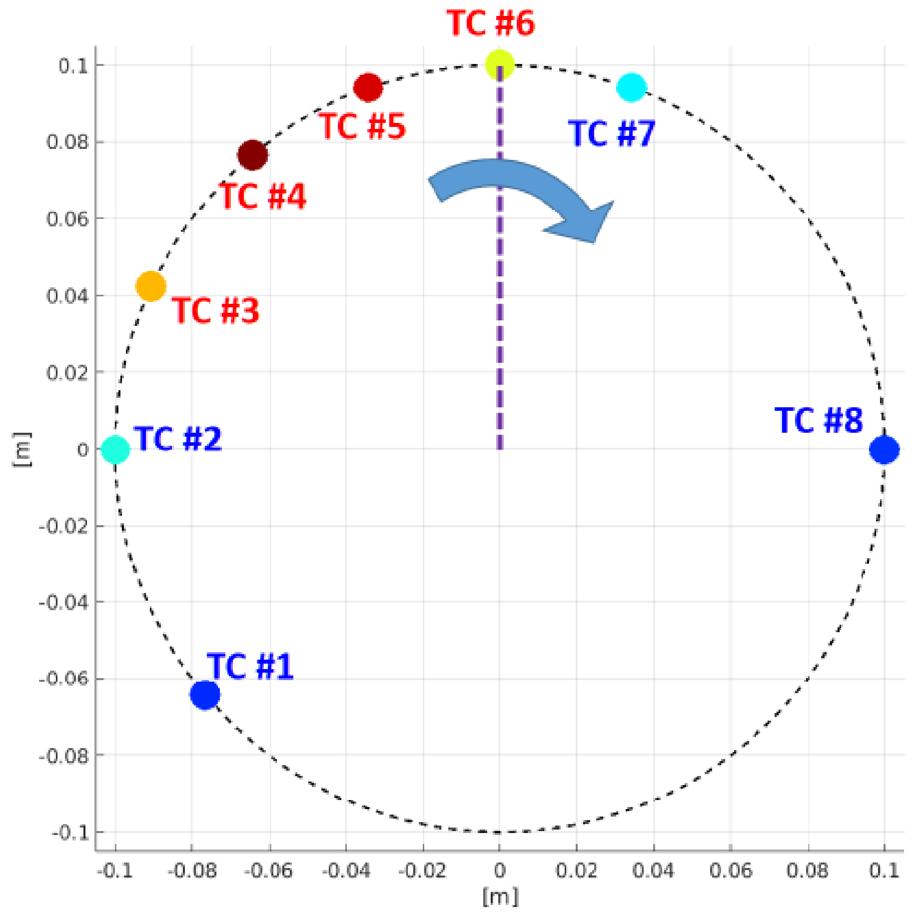
Experimental measurements of NBI duct heating in COMPASS

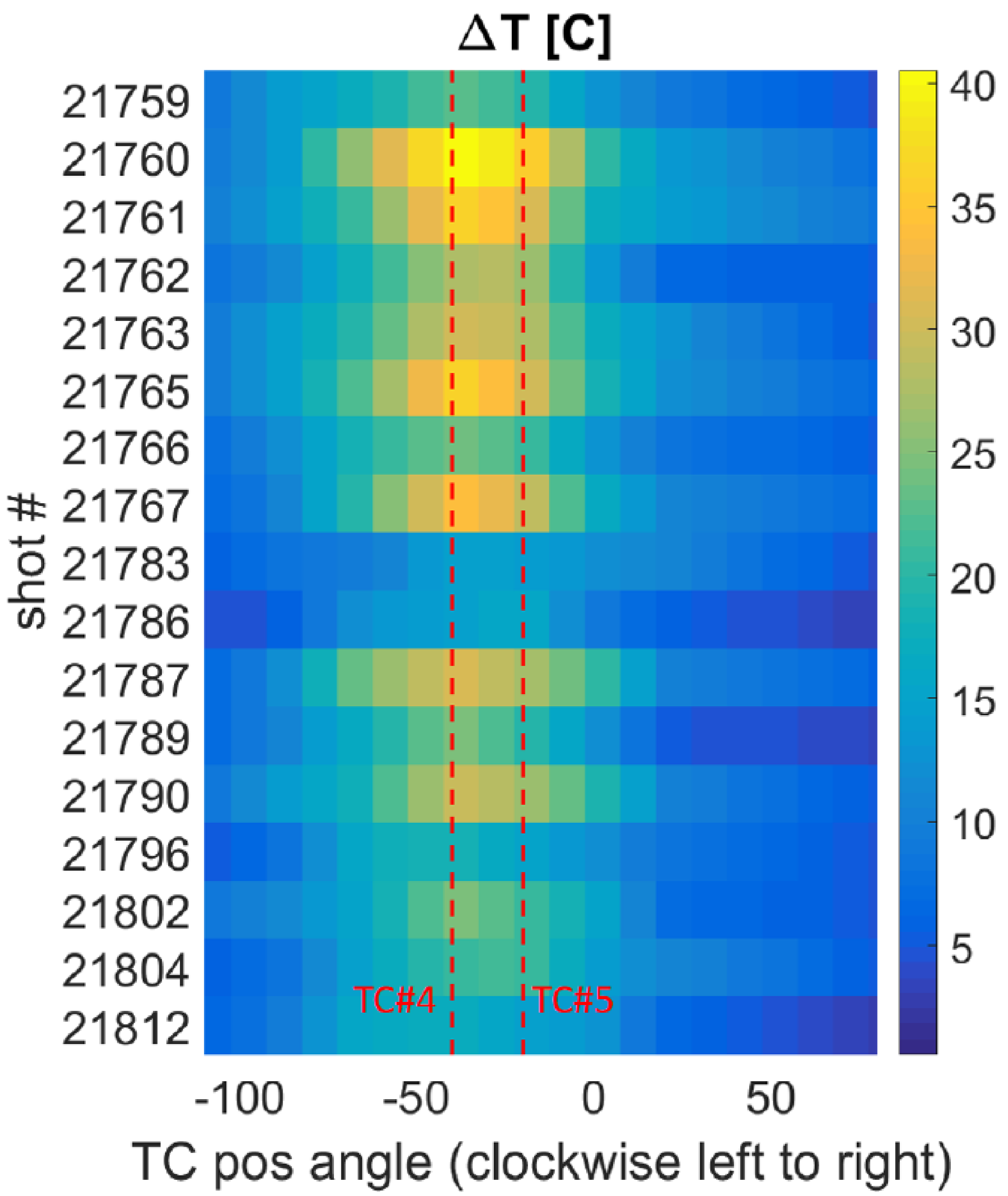




F. Jaulmes et al 2022 PPCF 64 125001

Experimental measurements of NBI duct heating in COMPASS

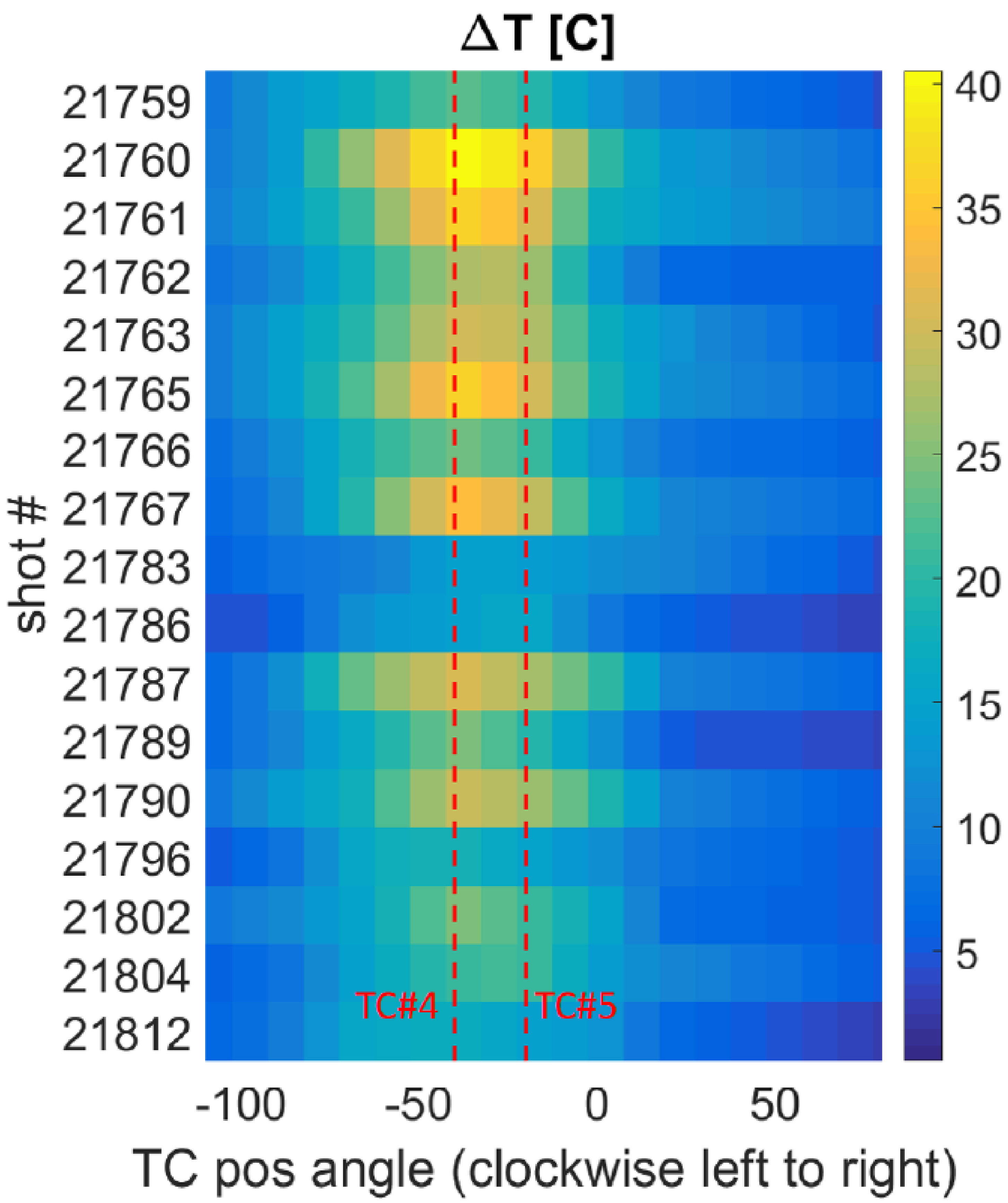




	B_t [T]	$\mathcal{E}_{inj}$ [keV]	P_{grid} [MW]	P_{effNBI} [MW]	Δt [ms]
21759	1.5	53	0.8	0.63	130
21760	1.5	58	1.0	0.77	230
21761	1.5	58	1.0	0.77	230
21762	1.5	59	1.0	0.77	180
21763	1.5	58	1.0	0.76	190
21765	1.5	59	1.0	0.76	220
21766	1.5	59	1.0	0.76	170
21767	1.5	58	1.0	0.75	230
21783	1.15	65	1.3	0.96	60
21786	1.15	66	1.4	1.0	60
21787	1.15	66	1.4	1.03	120
21789	1.15	66	1.4	0.99	90
21790	1.15	66	1.4	1.03	120
21796	1.15	55	0.7	0.55	100
21802	1.15	55	1.0	0.75	120
21804	1.38	60	1.0	0.75	110
21812	1.38	62	1.0	0.75	100

F. Jaulmes et al 2022 PPCF 64 125001

Experimental measurements of NBI duct heating in COMPASS



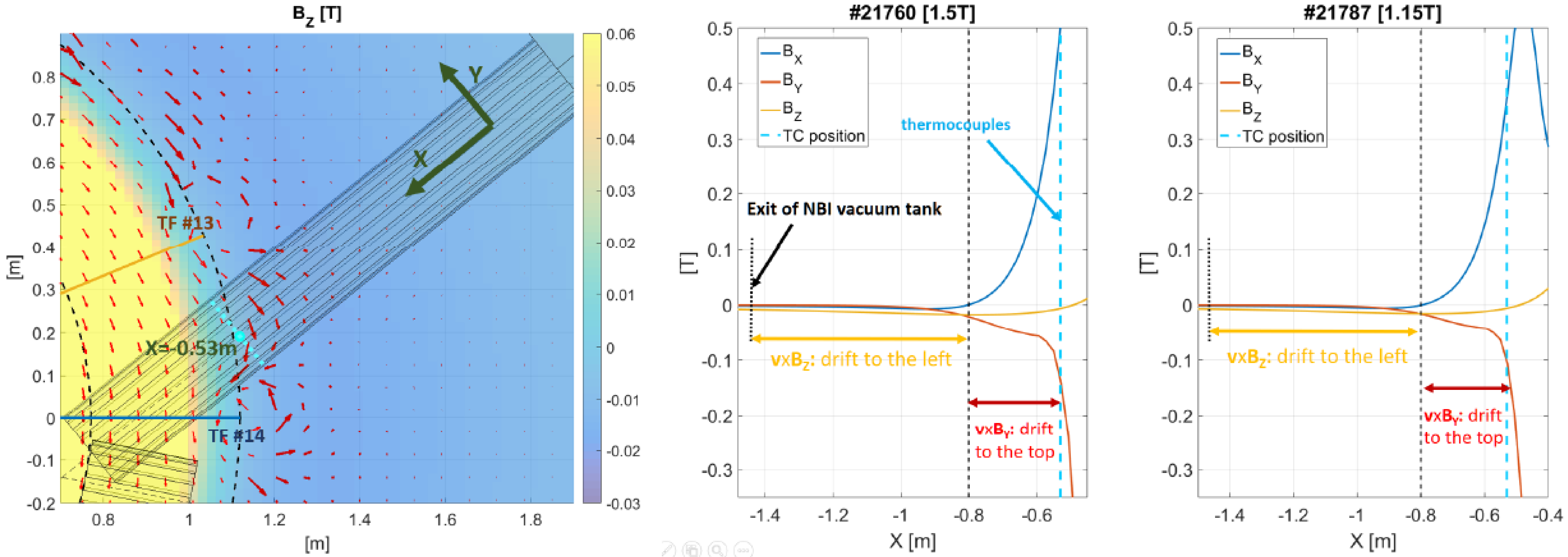
	B_t [T]	$\mathcal{E}_{inj}$ [keV]	P_{grid} [MW]	P_{effNBI} [MW]	Δt [ms]
21759	1.5	53	0.8	0.63	130
21760	1.5	58	1.0	0.77	230
21761	1.5	58	1.0	0.77	230
21762	1.5	59	1.0	0.77	180
21763	1.5	58	1.0	0.76	190
21765	1.5	59	1.0	0.76	220
21766	1.5	59	1.0	0.76	170
21767	1.5	58	1.0	0.75	230
21783	1.15	65	1.3	0.96	60
21786	1.15	66	1.4	1.0	60
21787	1.15	66	1.4	1.03	120
21789	1.15	66	1.4	0.99	90
21790	1.15	66	1.4	1.03	120
21796	1.15	55	0.7	0.55	100
21802	1.15	55	1.0	0.75	120
21804	1.38	60	1.0	0.75	110
21812	1.38	62	1.0	0.75	100

Strong localization along the duct suggests the influence of magnetic field dominates: fast ion losses dominate the power losses!

F. Jaulmes et al 2022 PPCF 64 125001

Modelling motions of fast ions inside the NBI duct

$$m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = Z_i e\left(\mathbf{v}\times\mathbf{B}\right)$$

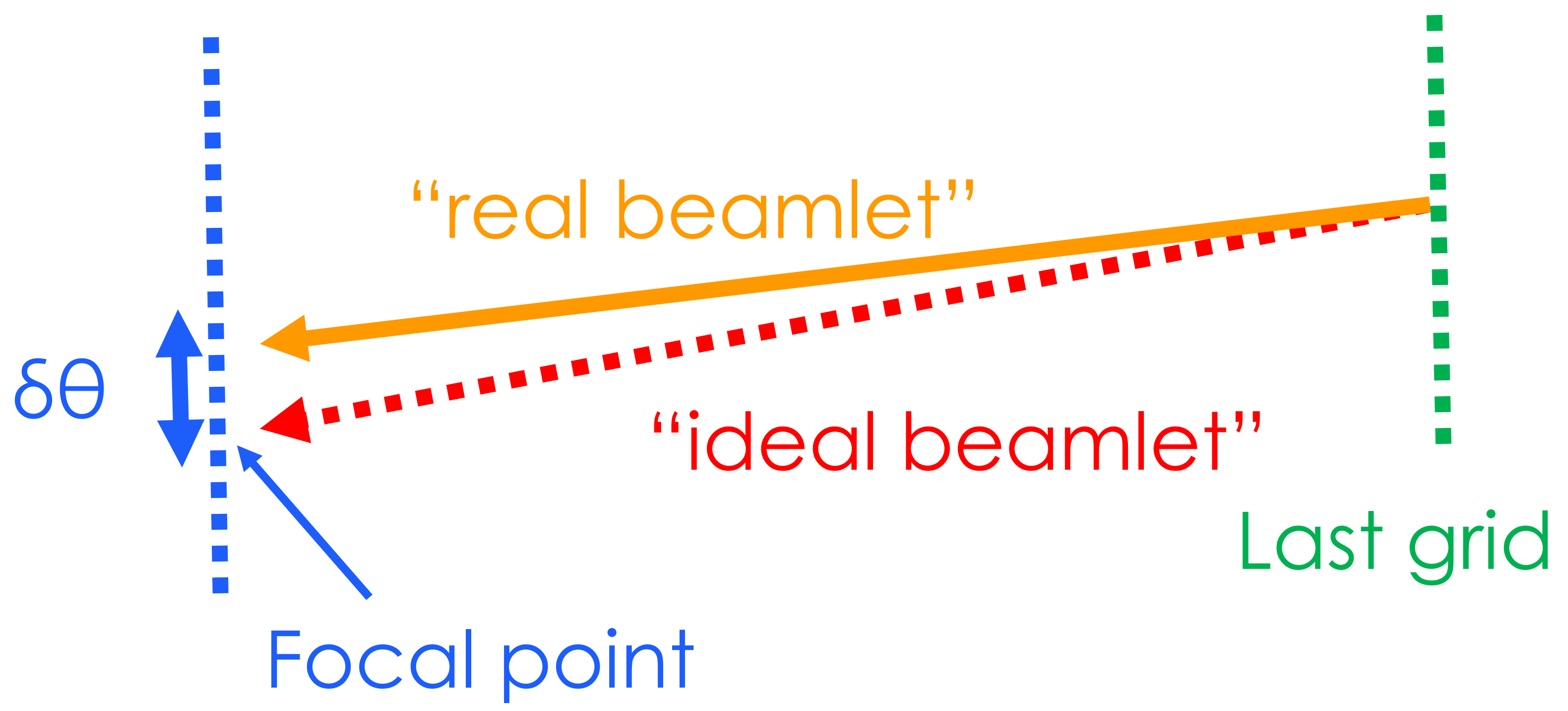


F. Jaulmes et al 2022 PPCF 64 125001

Trajectories of fast ions

$$f(t_g) = \frac{1}{\xi\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{t_g}{\xi}\right)^2\right)$$

$$t_g = \tan(\delta\theta) \simeq \delta\theta$$



$$\frac{\mathrm{d}\mathcal{P}_n}{\mathrm{d}l} = -\mathcal{P}_n n_0 \sigma(\mathcal{E})$$

$$\mathcal{P}_n = \exp\left(-\int n_0 \sigma(\mathcal{E}) \mathrm{d}l\right)$$

$$\mathcal{P}_i(l) = 1 - \mathcal{P}_n(l)$$

$$\mathcal{P}_i - \mathcal{R} = 0;$$

$$I = I_0 \left(\exp\left(-L n_0 \sigma_i\right)\right)$$

(uniform density)